Manjusha

Core Course PHYSICS

NUCLEAR PHYSICS AND PARTICLE PHYSICS

Dr. P. Sethumadhavan

1 NUCLEAR STRUCTURE AND RADIOACTIVITY

Introduction

Nucleus is the central core of every atom which occupies a very small volume. Within the nucleus there are Z positive charges. These positive charges repel due to electrostatic forces between them. So to hold these positive charges inside the nucleus, there must be some nuclear attractive force inside the nucleus to overcome the electrostatic force of repulsion. This nuclear force is the strongest force among the four known force. The nuclear force provides nuclear binding energies that are millions of times stronger than atomic binding energies.

There are many similarities between atomic and nuclear structure, which will make our study of the properties nucleus simple.

Similarities between nuclei and atoms

Both are governed by quantum mechanical laws. Both have ground state and excited states and emit radiations when they go from excited states to lower states. Both are labelled by their angular momentum quantum number.

Dissimilarities between nuclei and atoms

There are two major differences between the study of nuclei and atoms.

First one is that in atomic physics, the electrons in atoms experience the force provided by an external agent whereas in nuclear physics, nuclei can never experience an external force. In atomic physics we consider the interaction between nucleus and the electrons (single body problem) and the interaction between the electrons is often considered as a perturbation to the single body problem. In nuclear physics mutual interaction between the nuclear constituents is considered which provides the nuclear force. This interaction is a many body problem which cannot be treated as a single body problem.

The second difference between atomic and nuclear physics is that we cannot write the nuclear force in a simple form like the Coulomb force. There is no analytical expression describing the mutual interactions of constituents of nuclei.

Inspite of these difficulties we can learn about the properties of nuclei by study-

ing the interactions between different nuclei, the radioactive decay of nuclei and properties of nuclear constituents. In this chapter we discuss the properties of nuclei.

This study is of great importance because of several reasons. One is that the very existence of the various elements is due to the ability of nuclei to possess multiple electric charges. The second one is that energy involved in almost processes are due to nuclear reactions and transformations. Above all the liberation of nuclear energy in reactors and weapons play a crucial role in our everyday life.

Nuclear constituents

The work of Rutherford, Bohr and etal revealed that the entire positive charge of atom is concentrated in a small of space (10^{-15} m) called nucleus. The nucleus in an atom of atomic number Z has charge of Ze. The mass of these charges provides 99.9% of its atomic mass. At that time it was also known that the masses of the atoms were integers within an accuracy of 0.1%, we call this integer A the mass number. It was therefore reasonable to suppose that nuclei are composed of a mass number A. At that time it was known that the mass of the proton is nearly 1u (atomic mass unit). If the mass is Au, we can very well say that the nucleus contains A proton.

Such a nucleus would have a nuclear charge of Ae rather than Ze, because $A > Z$ for all atoms heavier than hydrogen. This model gives two much positive charge to the nucleus. This difficulty was removed by bringing another model called proton-electron model. In this model they postulated that the nucleus also contained (A-Z) electrons called nuclear electrons. Under these assumptions the nuclear mass would be A times the mass of the proton, because the mass of the electrons is negligible. Then the nuclear charge would be Ae + (A − Z) (−e) = Ze. This is in agreement with the experimental results. The postulate of nuclear electrons was supported by the fact that certain radioactive nuclear spontaneously emit electrons, a phenomenon called beta decay. However there are strong evidences against the idea of nuclear electrons. They are

1. Nuclear size

When an electron is confined to a box of nuclear dimensions it possesses an energy more than 19 MeV according to uncertainty principle whereas electrons emit-ted during beta decay have energies 2-3 MeV.

According to uncertainty principle

$$\Delta x \Delta p \approx \hbar, \text{ here } \Delta x = 10^{-14} \, m$$

$$\therefore \qquad \Delta p \approx \frac{\hbar}{\Delta x} \simeq \frac{10^{-34}}{10^{-15}} = 10^{-20}$$

Using $\quad E = \sqrt{p^2 c^2 + m_0^2 c^4} = \sqrt{(10^{-20})^2 \times (3 \times 10^8)^2 + (9 \times 10^{-31})^2 (3 \times 10^8)^4}$

$$= 3 \times 10^{-12} \, J = 18.75 \ MeV$$

2. Nuclear spin

Protons and electrons are fermions with spins $\frac{1}{2}$. Thus nucleus with even number of protons plus electrons should have zero or integral spin, those with an odd number of protons plus electrons should have half integral spins. But it has been experimentally found that it is not true. For example a deuterium nucleus consists of two protons and an electron, its nuclear spin should be either $\frac{1}{2}$ or $\frac{3}{2}$. But the spin observed is one.

3. Magnetic moment

It has been found that the magnetic moment of proton is only about 0.15 present that of electrons. If nuclear electrons exist, the magnetic moment of a nucleus is of the order of magnitude of that of the electron. But the observed magnetic moment of the nucleus is found to be of the order of magnitude of protons.

Because of these and other reasons the idea of nuclear electrons was abandoned. The resolution of this dilemma came in 1932 with the discovery of neutron by James Chadwick. The discovery of neutron with atomic mass unit roughly the same as the proton (actually 0.14% more massive than proton) without electric charge led to the assumption that every atomic nucleus consists of protons and neutrons. This hypothesis was used for the first time as the basis of a detailed theory of the nucleus by Heisenberg in 1932. According to proton-neutron model, the total number of elementary particles in the nucleus, protons and neutrons together, is equal to the mass number A of the nucleus. The number of protons is given by Z and the number of neutrons is A-Z.

The proton and neutron together are called as nucleus. Some properties of nucleons are given below.

Table 1.1: Properties of nucleons

Name	Symbol	Charge	Mass	Rest energy	Spin
Proton	p	+e	1.007276 u	938.28 MeV	$\frac{1}{2}$
Neutron	n	0	1.008665 u	939.57 MeV	$\frac{1}{2}$

Note: Recall that one atomic mass unit, $1u = 931.5$ MeV/c^2.

i.e. to convert one atomic mass unit into MeV multiply by 931.5

A nuclide is a specific nucleus of an atom which is characterised by its atomic number Z and mass number A. It is represented as $_Z X^A$ or $_Z^A X$, where X is the chemical symbol of the species.

Each nuclear species with a given Z and A is called a nuclide. Each Z characterises a chemical element with symbol X. For example, Al for aluminium ($Z = 13$), Ca for calcium ($Z = 20$). Nuclear mass is roughly the sum of its constituents proton and neutron masses (the slight difference being the binding energy of the nucleus). The nuclear charge is +e times the number (Z) of protons ($1e = 1.6 \times 10^{-19}$ C).

The chemical properties of an atom are determined by its electron configuration because the number of electrons and protons are equal in neutral atom. The chemical properties are essentially determined by Z. The dependence of the chemical properties on N is negligible.

Nuclei with the same Z but different A are called isotopes. For example $_1^1 H$ (ordinary hydrogen), $_1^2 H$ (deuterium) and $_1^3 H$ (tritium) are isotopes of hydrogen. Another example is the four isotopes of carbon: $_6^{11}C$, $_6^{12}C$, $_6^{13}C$ and $_6^{14}C$.

Nuclides with the same neutron number are called isotones

For example: $_6^{14}C$, $_7^{15}C$, $_8^{16}C$ and $_9^{17}C$.

Nuclide with the same mass number are called isobars.

For example: $_6^{16}C$, $_7^{16}N$, $_8^{16}O$ and $_9^{16}F$.

Example 1

Give the proper isotopic symbols for (a) the isotope of fluorine with mass number 19 (b) an isotope of gold with 120 neutrons (c) an isotope of mass number 107 with 60 neutrons.

Solution

(a) Fluorine has Z = 9, so the symbol is $^{19}_{9}$F.

(b) Gold has Z = 79. So N + Z = 120 + 79 = 199. Thus the symbol is $^{199}_{79}$Au.

(c) Here A = 107, N = 60 ∴ Z = A − N = 107 − 60 = 47. Z = 47 is silver. Thus the symbol is $^{107}_{47}$Ag.

Nuclear sizes and shapes

Atoms have hard surfaces thereby they have well defined radii even though they are probabilistic. But nuclei do not have a hard surface hence no easily definable radius. At the same time different types of experiments often reveal different values of the radius for the same nucleus.

Several experiments had been conducted to find the variation of nuclear density with the radius of the nuclei. This is given in figure below.

From the graph two things can be inferred.

1. Nuclear density is independent of mass number A.

2. Nuclear density is almost uniform for all nuclei.

Figure 1.1: The radial dependence of the nuclear charge density

From this important clue, we can arrive at the relation between the radius and mass number of nuclei.

$$\text{density } (\rho) = \frac{\text{Mass}}{\text{Volume}} = \frac{A}{\frac{4}{3}\pi R^3} = \text{constant}$$

Assumed that nucleus is a sphere of radius R.

i.e.,
$$A = \text{constant} \cdot \frac{4}{3}\pi R^3$$

i.e.,
$$A \propto R^3$$

This relationship is expressed in inverse form as

$$R \propto A^{1/3}$$

$$R = R_0 A^{1/3} \qquad \qquad \dots\dots (1)$$

or

where R_0 is a constant whose value can be easily calculated from the above graph. For example, for ^{12}C radius is 2.7 fm from the graph and A = 12, use this in equation (1), we get

$$R_0 = \frac{R}{A^{1/3}} = \frac{2.7 fm}{(12)^{1/3}} \simeq 1.2 fm$$

As A is different for different atoms atomic nuclei of different atoms have different sizes.

Note: fm is the femtometer. 1 fm = 1 femtometer is also called as fermi in honour of the Italian physicist Fermi.

Example 2

Find the density of $_6C^{12}$ nucleus.

Solution

We have density, $\rho = \dfrac{\text{Mass}}{\text{Volume}}$

i.e., $\qquad \rho = \dfrac{m}{\dfrac{4}{3}\pi R^3} = \dfrac{m}{\dfrac{4}{3}\pi (R_0 A^{\frac{1}{3}})^3}$

$$m = 12u = 12 \times 1.66 \times 10^{-27} kg$$

$$R_0 A^{\frac{1}{3}} = 1.2 \times 10^{-15} \times (12)^{\frac{1}{3}} = 2.7 \times 10^{-15} m$$

these values define the mean radius the point at which the density falls to half the central value. — from fig (1.1)

$$\therefore \qquad \rho = \frac{12 \times 1.66 \times 10^{-27}}{\frac{4}{3} \cdot \pi \times (2.7 \times 10^{-15})^3} = 2.4 \times 10^{-17} kgm^{-3}$$

Example 3

Find the nuclear radius of (a) ^{197}Au (b) ^{20}Ne and (c) ^4He

Solution

(a) $A = 197, R_0 = 1.2 fm$

Using $\qquad R = R_0 A^{1/3}$

$$R = 1.2 \times (197)^{1/3} = 6.98\,\text{fm}$$

(b) $A = 20$, $R = 1.2\,\text{fm}$

$$R = 1.2 \times (20)^{1/3} = 3.26\,\text{fm}$$

(c) $A = 4$, R $R_0 = 1.2\,\text{fm}$

$$R = 1.2 \times (4)^{1/3} = 1.90\,\text{fm}$$

Nuclear masses and binding energies

Consider a proton and an electron at rest separated by a large distance. The total energy of this system (proton and electron) is the total rest energy of the two particles.

i.e., total energy of the system $= m_p c^2 + m_e c^2 \qquad \qquad \dots (2)$

Now suppose that these two particles bring together to form a hydrogen atom in its ground state. In the process of bringing energy is liberated in the form of photons, say it is 13.6 eV. Now the total energy of the system is rest energy of the hydrogen atom $m(H)c^2$ plus the energy liberated (13.6 eV).

i.e., total energy of the system $= m(H)c^2 + 13.6\text{ eV} \qquad \qquad \dots (3)$

According to law of conservation of energy, for an isolated system total energy is conserved. i.e., eq (2) = eq (3)

Thus we get

$$m_p c^2 + m_e c^2 = m(H)c^2 + 13.6\text{ eV}$$

or $\qquad\qquad m_p c^2 + m_e c^2 - m(H)c^2 = 13.6\text{ eV} \qquad \qquad \dots (4)$

This equation says that the rest energy of the combined system $m(H)c^2$ is less than the rest energy of its constituents by an amount 13.6 eV. This energy difference is called the binding energy (E_b) of the atom. Thus binding energy can be defined as follows.

Binding energy is the extra energy that we obtain when we assemble an atom from its constituents or it is the energy that we must supply to dissemble the atom into its constituents.

Equation (4) can be written as

$$[m_p + m_e - m(H)]c^2 = 13.6 \text{ eV}$$

The term inside the square bracket is the difference mass of an atom and its constituents is called the mass defect denoted by Δm.

i.e.,
$$\Delta mc^2 = 13.6 \text{ eV}$$

In general
$$\Delta mc^2 = E_b \qquad \qquad \dots\dots (5)$$

Thus binding energy E_b is the energy equivalent of the mass defect of the atom. In a similar way we can calculate nuclear binding energies.

Example: Consider the nucleus of deutirium ${}_1^2H$, which composed of one proton and one neutron. The nuclear binding energy of deuterium is the difference between the total rest energy of the constituents and the rest energy of their combination:

$$E_b = m_n c^2 + m_p c^2 - m_D c^2 \qquad \qquad \dots\dots (6)$$

where m_D is the mass of the deuterium nucleus.

Usually we do calculations by using standard tables of atomic masses. Thus m_p is replaced by mass of the hydrogen atom $m({}_1^1H)$ and m_D is replaced by deutron mass $m(D)$

$$m({}^1H_1)c^2 = m_p c^2 + m_e c^2$$

or
$$m_p c^2 = m({}_1^1H)c^2 - m_e c^2$$

$$m(D)c^2 = m_D c^2 + m_e c^2$$

or
$$m_D c^2 = m(D)c^2 - m_e c^2$$

Replacing $m_p c^2$ and $m_D c^2$ from equation (6) by the above relations we get

$$E_b = m_n c^2 + m({}_1^1H)c^2 - m_e c^2 - (m(D))c^2 - m_e c^2)$$

$$E_b = m_n c^2 + m({}_1^1H)c^2 - m(D)c^2$$

or
$$E_b = [m_n + m({}_1^1H)c^2 - m(D)]c^2$$

$$\dots\dots (7)$$

Nuclear masses are expressed in atomic mass units. Substituting the values of m_n, $m(H)$ and $m(D)$, we get

$$E_b = [1.008665\,u + 1.007825\,u - 2.014102\,u]c^2$$

$$E_b = 0.002388\,u\,c^2$$

Using $1\,u = 931.5\,\dfrac{MeV}{c^2}$

$$E_b = 0.002388 \times 931.5\,\dfrac{MeV}{c^2}\,c^2$$

$$E_b = 0.002388 \times 931.5\;MeV$$

$$E_b = 2.224422\;MeV$$

Example: In the case of $^{16}_{8}O$, we have 8 neutrons and 8 protons.

$$\text{Mass of 8 neutrons} \quad = 8\,m_n$$

$$= 8 \times 1.008665\;u$$

$$= 8.06932\;u$$

$$\text{Mass of 8 protons} \quad = 8m(^{1}_{1}H) = 8 \times 1.007825\;u$$

$$= 8.0626\;u$$

$\therefore$　Total mass of constituents　$= 8.06932\,u + 8.0626\,u$

$$= 16.13192\,u$$

Mass of $^{16}_{8}O$ atom　$= 16\,u$

$\therefore$　The mass defect,　$\Delta m \quad = 16.13192\,u - 16\,u$

$$\Delta m \quad = 0.13192\,u$$

$\therefore$　Thus binding energy E_b　$= \Delta m c^2 = 0.13192\,u c^2$

Using $1u$　$= 931.5\,\dfrac{MeV}{c^2}$

$$E_b \quad = 0.13192 \times 931.5\;MeV$$

$$E_b \quad = 122.88348\;MeV$$

i.e., the binding energy of oxygen nucleus is 122.88348 MeV.

Now we can generalise this process to calculate the binding energy of a nucleus X of mass number A with Z protons and N neutrons. Let m_X represent the mass of this nucleus.

The binding energy of a nucleus $^A_Z X$ is given by

$$E_b = N m_n c^2 + Z m_p c^2 - m_X c^2 \qquad \dots (8)$$

We do this calculation by using standard tabulated atomic masses. For this we replace the nuclear mass m_X with its corresponding atomic mass $m(^A_Z X)$.

$$m(^A_Z X) c^2 = m_X c^2 + Z m_e c^2 - e_b$$

where e_b represents the total binding energy of all electrons in this atom. Usually $m_X c^2$ is of the order of 10^9 to 10^{11} eV, $Z m_e c^2$ is of the order of 10^6 to 10^8 eV and e_b is of the order of 1 to 10^5 eV. Thus e_b is very small compared with other two energies we can very well neglect e_b.

Thus we have

$$m(^A_Z X) c^2 = m_X c^2 + Z m_e c^2$$

or

$$m_X c^2 = m(^A_Z X) c^2 - Z m_e c^2 \qquad \dots (9)$$

Now replace proton mass m_p with atomic mass of hydrogen $m(^1_1 H)$.

$$m(^1_1 H) c^2 = m_p c^2 + m_e c^2$$

or

$$m_p c^2 = m(^1_1 H) c^2 - m_e c^2 \qquad \dots (10)$$

substituting equations 9 and 10 in equation 8 we get

$$E_b = N m_n c^2 + Z(m(^1_1 H) c^2 - m_e c^2) - (m(^A_Z X) c^2 - Z m_e c^2)$$

$$E_b = N m_n c^2 + Z m(^1_1 H) c^2 - m(^A_Z X) c^2$$

$$E_b = [N m_n + Z m(^1_1 H) - m(^A_Z X)] c^2 \qquad \dots (11)$$

This is the general expression for the binding energy of a nucleus $^A_Z X$

Therefore the binding energy per nucleon for a given nucleus is $\dfrac{E_b}{A} = \bar{E}_b$

i.e., $$E_b = \frac{[Nm_n + Zm(^1H_1) + m(^A_ZX)]c^2}{A} \qquad \ldots \ldots (12)$$

It may be noted that masses are expressed in atomic masses and automatically electron masses cancel out.

Example 4

Calculate the binding energy of tin nucleus $(^{120}_{50}Sn)$. Given: atomic mass of $^{120}Sn = 119.\,9022$ u, mass of hydrogen atom $m(^1_1H) = 1.007825u$, mass of neutron $m_n = 1.008665$ u.

Solution

Binding energy of nucleus, $E_b = [Nm_n + Zm(^1_1H) - m(^A_ZX)]c^2$

Tin $(^{120}_{50}Sn)$ has 70 neutrons and 50 protons.

Thus, $E_b = [70 \times 1.008665\,u + 50 \times 1.007825\,u - 119.9022\,u]c^2$

$E_b = [70.60655\,u + 50.39125\,u - 119.9022\,u]c^2$

$E_b = 1.0956\,u\,c^2$

Using $1u = 931.5\dfrac{MeV}{c^2}$

$\therefore \qquad E_b = 1.0956 \times 931.5\ MeV$

$E_b = 1020.5514\ MeV$

Example 5

Find the total binding energy and the binding energy per nucleon for

a) $^{208}_{82}Pb$ and b) $^{90}_{40}Zr$

$m_n = 1.008665\,u,\ \ m(^1_1H) = 1.007825\,u,\ \ m(Pb) = 207.976652\,u$ and $m(Zr) = 89.904704\,u.$

Solution

Binding energy, $E_b = [Nm_n + Zm(^1_1H) - m(^A_ZX)]c^2$

a) For lead $\qquad E_b = [Nm_n + Zm(_1^1H) - m(Pb)]c^2$

Lead nuclei containing 126 neutrons and 82 protons

$\therefore \quad E_b = [126 \times 1.008665\,u + 82 \times 1.007825\,u - 207.976652\,u]c^2$

$E_b = [127.09179\,u + 82.64165\,u - 207.976652\,u]c^2$

$E_b = 1.756788\,u\,c^2$

$E_b = 1.756788 \times 931.5\ \text{MeV}$

$E_b = 1636.448022\ \text{MeV}$

$\therefore \quad$ Binding energy per nucleon $\quad \bar{E}_b = \dfrac{E_b}{A} = \dfrac{1636.448022}{208}$

$$\bar{E}_b = 7.868\ \text{MeV per nucleon.}$$

b) For Zirconium nucleus

$E_b = [Nm_n + Zm(_1^1H) - m(Zr)]c^2$

Zirconium nucleus has 50 neutrons and 40 protons.

$E_b = [50 \times 1.008665\,u + 40 \times 1.007825\,u - 89.904704\,u]c^2$

$E_b = [50.43325\,u + 40.313\,u - 89.904704\,u]c^2$

$E_b = 0.841546\,uc^2$

$E_b = 0.841546 \times 931.5\,\text{MeV}$

$E_b = 783.9\,\text{MeV}$

$\therefore \quad$ Binding energy per nucleon,

$$\bar{E}_b = \frac{E_b}{A} = \frac{783.9}{90} = 0.71\,\text{MeV per nucleon.}$$

Binding energy curve

A plot of binding energy per nucleon $\left(\bar{B} = \dfrac{BE}{A} \right)$ as a function of mass number A

for various stable nucleus is called a binding energy curve. Figure given above shows the binding energy curve. Following points may be noted from the curve.

(i) The greater the binding energy per nucleon the more stable is the nucleus. The graph has its maximum value of 8.8 MeV/nucleon for $^{56}_{26}$Fe i.e., number of nucleon is 56. This is the most stable nucleus, since maximum energy is needed to pull a nucleon away from it.

(ii) If each nucleon interacted with every nucleon, there would be (A-1) interactions for each nucleon, and the binding energy would be proportional to (A-1) rather than constant. Instead, there is a fixed number of interactions per nucleon, because each nucleon is attracted to its nearest neighbours. Such a situation also leads to a constant nuclear density. If the binding energy per nucleon were instead proportional to the number of nucleons, then the radius would be approximately constant, as in the case of atoms.
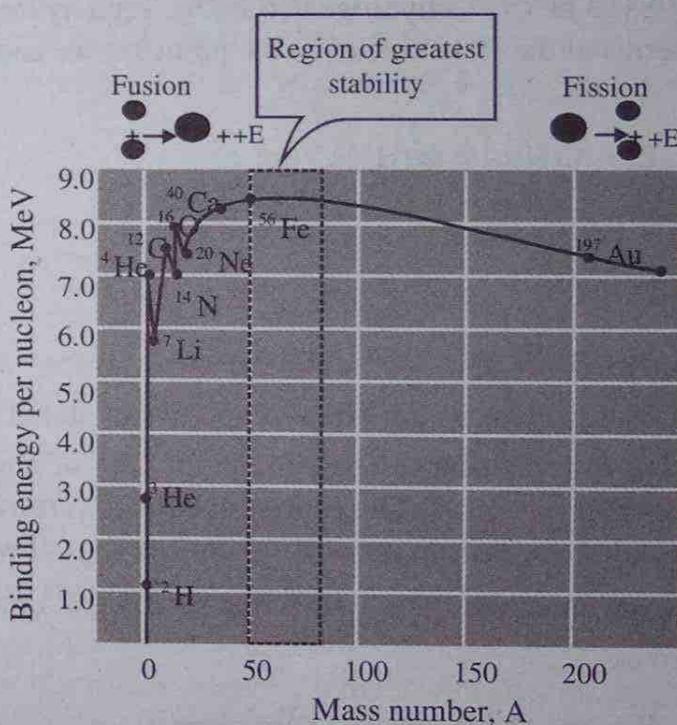

Figure 1.2

(iii) If we split a heavy nucleus into two medium sized ones, each of the new nuclei will have more binding energy per nucleon than the original nucleus did. The extra energy will be given off.

Binding energy curve droops at high mass number which tells us that nucleons are more tightly bound when they are assembled into two middle-mass nuclei rather than into a single high- mass nucleus. Energy is released in the nuclear fission of a single massive nucleus into two smaller fragments.

(iv) The drooping of the binding energy curve at low mass number tells us that energy will be released if two nuclei of small mass number combine to form a single middle mass nucleus. New nucleus has more binding energy per nucleon.

For instance, if two $_1^2\text{H}$ deuterium nuclei combine to form $_2^4\text{He}$ helium nucleus, over 23 MeV is released. Such process is called **nuclear fusion**.

Proton and neutron separation energies

Firstly we recall the definition of ionisation energy (E_i) of an atom. **It is the amount of energy need to remove an electron from an atom**. For example, the ionisation energy of hydrogen atom in the ground state is +13.6 eV. If we add the ionisation energy E_i(13.6eV) to a hydrogen atom, we get a hydrogen ion (H⁺) and a free electron. In terms of the rest energies of the particles, we can write this process as

$$E_i + m(H)c^2 = m(H^+)c^2 + m_e c^2$$

In general, for any atom X

$$E_i + m(X)c^2 = m(X^+)c^2 + m_e c^2$$

or
$$E_i = m(X^+)c^2 + m_e c^2 - m(X)c^2$$

The ionisation energy gives us important properties of atoms.

A process similar to ionisation can be done in the case of nucleus: Removing a proton or a neutron from a nucleus. **The energy needed to remove the least tightly bound proton is called the proton separation energy (S_p)**. If we add energy S_p to a nucleus $_Z^A X$, we get a nucleus $_{Z-1}^{A-1} X'$ and a free proton (p). In mathematical form this can be written as

$$S_p + m(_Z^A X)c^2 = m\left(_{Z-1}^{A-1}X'\right)c^2 + m_p c^2$$

or
$$S_p = m\left(_{Z-1}^{A-1}X'\right)c^2 + m_p c^2 - m(_Z^A X)c^2$$

In terms of atomic mass units, this can be written as

$$S_p = m\left(_{Z-1}^{A-1}X'\right)c^2 + m(_1^1 H)c^2 - m(_Z^A X)c^2$$

or
$$S_P = \left[m\left(^{A-1}_{Z-1}X'\right) + m(^1_1H) - m(^A_ZX) \right] c^2 \qquad \dots (13)$$

This is the expression for proton separation energy. (see also example 6).

Similarly if we add the neutron separation energy (S_n) to a nucleus ^A_ZX, we get the nucleus $^{A-1}_{Z-1}X$ and free neutron.

i.e.
$$S_n + m(^A_ZX)c^2 = m\left(^{A-1}_{Z-1}X\right)c^2 + m_n c^2$$

or
$$S_n = \left[m\left(^{A-1}_{Z-1}X\right) + m_n - m(^A_ZX) \right] c^2 \qquad \dots (14)$$

This is the expression for neutron separation energy.

Proton and neutron energies are in the range of 5 – 10 MeV, this is the same as the average binding energy per nucleon. (see also example 7).

The proton and neutron separation energies play a role in nuclei similar to ionisation energy in atoms. Figure below shows a plot of the neutron separation energies of nuclei with a valence neutron from Z = 36 to Z = 62. Following points may be noted from the graph
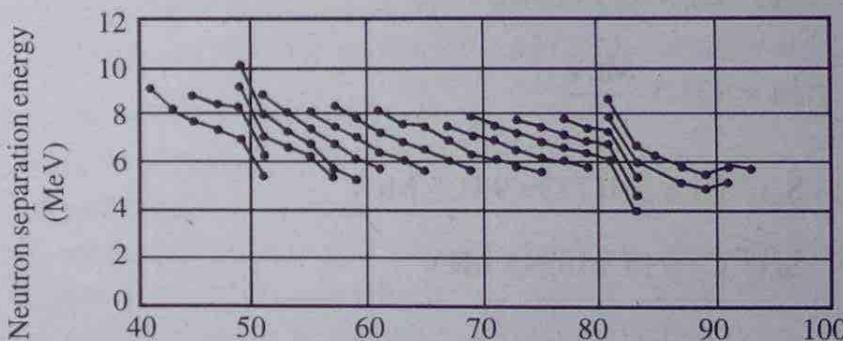
Figure 1.3: The neutron separation energy. The lines connected isotopes of the same element that have an odd neutron starting on the left at Z = 36 and ending on the right at Z = 62

As we add neutrons the neutron separation energy decreases smoothly except near N = 50 and N = 82, where there are more sudden decreases in the separation energy. In analogy with atomic physics these sudden decreases are associated with the filling of shells. The motions of neutrons and protons in the nucleus are described in terms shell structure that is similar to that of atomic shells and when a neutron or proton is placed into a new shell it is less tightly bound and its separation energy decreases. The neutron separation data indicate that there are closed neutron shells at N = 50 and N = 82. The figure above thus provides an important information about the shell structure of the nuclei.

Example 6

Find the proton separation energy of ^{12}C. Given that $m(^1_1H) = 1.007825\,u$, $m(^{11}B) = 11.009305\,u$ and $m(^{12}C) = 12.000000\,u$.

Solution

If we add proton separation energy to a nucleus $^A_Z X$ it becomes $^{A-1}_{Z-1}X' + p$.

In the case of carbon it becomes boron (B)

Separation energy of proton in general is given by

$$S_p = \left[m\left(^{A-1}_{Z-1}X'\right) + m(^1_1H) - m(^A_Z X) \right] c^2$$

$$S_p(^{12}C) = \left[m(^{11}B) + m(^1_1H) - m(^{12}C) \right] c^2$$

Substituting the m values given, we get

$$S_p(^{12}C) = \left[11.009305\,u + 1.007825\,u - 12.000000\,u \right] c^2$$

$$S_p(^{12}C) = 0.01713\,u\,c^2$$

Using $1u = 931.5\,\dfrac{MeV}{c^2}$

$$S_p(^{12}C) = 0.01713 \times 931.5\ MeV$$

$$S_p(^{12}C) = 15.956595\ MeV$$

Example 7

Find the neutron separation energy of ^{57}Fe. Given that $m_n = 1.008665\,u$, $m(^{56}Fe) = 55.934937\,u$ and $m(^{57}Fe) = 56.935394\,u$

Solution

The neutron separation energy is given by

$$S_n = \left[m\left(^{A-1}_{Z}X\right) + m_n - m\left(^A_Z X\right) \right] c^2$$

$$S_n(^{57}Fe) = \left[m(^{56}Fe) + m_n - m(^{57}Fe) \right] c^2$$

Substituting the values of m, we get

$$S_n(^{57}\text{Fe}) = [55.934937\,u + 1.008665\,u - 56.935394\,u]c^2$$

$$S_n(^{57}\text{Fe}) = 0.08208\,u\ c^2$$

Using $\qquad 1\,u = 931.5\dfrac{\text{MeV}}{c^2}$

$$S_n(^{57}\text{Fe}) = 0.08208 \times 931.5\ \text{MeV}$$

$$S_n(^{57}\text{Fe}) = 7.645752\ \text{MeV}$$

Nuclear models

To explain the behaviour and properties of nuclei several models have been proposed so far. Unlike the case of the atom, physicists however, still donot have a very clear understanding of the details of nuclear forces or the way the nucleons interact with each other. This lack of understanding has made them propose very many different models each designed to explain a particular category of nuclear phenomena or nuclear properties. These models even tend to contradict each other. This has led to considerable confusion and attempts are constant on to clear this confusion and get a model that incorporates the essential details of these very many models.

The nuclear models proposed so far can be categorized into two broad types. They are (1) the strong interaction models and (2) the independent particle models.

The strong interaction models

These models are based on the assumption that the nucleons, in a nucleus, are strongly coupled together. The one model of this category is liquid drop model. This model explains the phenomenon of nuclear fission. It has also been used to arrive at the semi empirical mass formula which enables us to calculate the nuclear masses in a satisfactory way (from the knowledge of binding energy).

The independent particle models

These models are based on the assumption that there exists a common nuclear potential within the nucleus and all the nucleons move nearly independently within this common nuclear potential. The most successful model of this category is the shell model. This model helps us to understand the periodicity observed in many properties of the nuclei.

Here we shall deal with the liquid drop model and the shell model.

The liquid drop model

The liquid drop model of the nucleus was first suggested by Bohr. He along with wheeler used it to explain the essential details of the phenomenon of nuclear fission.

This explanation has been the most significant achievement of this model but it failed to understand other nuclear properties. It is now a more or less obsolete model.

Because of some striking similarities between a liquid drop and a nucleus, this model was proposed. We give some of the similarities below.

1. Small drops of a liquid are known to acquire a spherical shape. We know that this is because of the symmetrical surface tension forces acting towards the centre of the drop. The gravitational forces, acting vertically downwards, tend to flatten out the drop. Ultimately, it is the balance between these two forces which decides the shape of the drop.

    A nucleus also has two kinds of opposing forces acting within it. On the one hand there are the (short ranged and very strong) attractive nuclear forces that try to keep the nucleus intact. On the other hand, there are the repulsive electrostatic forces between the protons contained in the nuclei. It is again a balance between these two opposing forces which decides the stability level of a nucleus. For light stable nuclei, the attractive nuclear forces dominate the other repulsive (disruptive) forces. Such nuclei, therefore, tend to be spherical in shape.

2. For a spherical drop, the density is known to be independent of its volume. This also implies that the radius must vary as the cube root of its mass. It is known that the density of nuclear matter is (nearly) constant for all nuclei and nuclear radii vary as the cube root of their mass numbers. We know that, unlike different nuclei, the densities of different liquids are different. However, there is still sufficient reason to think of a nucleus as if it were akin to a spherical drop.

3. The molecules of a liquid are known to be in continuous random thermal motion. We use this to explain the phenomenon of continuous evaporation of a liquid and the increase in the rate of this evaporation with an increase in the temperature of the liquid. If we were to think of a similar random motion for the nucleons inside a nucleus, we could think of phenomenon like spontaneous emission of α-particles as essentially similar to the evaporation of molecules from the surface of a liquid.

According to this model the nucleus is regarded as a collection of neutrons and protons forming a droplet of incompressible liquid which behaves in some ways like a liquid drop.

The liquid drop model is based on two properties of nuclei.

(i) The interior mass densities are same

(ii) Their total binding energies are approximately proportional to their masses. i.e.,

$$\frac{\Delta E_b}{A} = \text{a constant (except for small A)}.$$

The semiempirical mass formula

This formula helps us to calculate the mass of a given nucleus from a knowledge of its mass number A and its atomic number (Z). This was derived by Von Weizsacker in 1935.

His arguments were based on the assumption of a liquid drop type model for a nucleus.

By simple logic, we expect the mass, M of a nucleus of atomic number Z and mass number A to be

$$M = Zm_p + (A - Z)m_n$$

Here m_p and m_n stand for the masses of the proton and neutron respectively. Precise measurements of nuclear masses however indicated that the observed nuclear masses are always less than their ideally expected mass given by the above formula. We now believe that this lost mass gets converted into energy as per the Einstein mass energy relation $(E = mc^2)$. This energy equivalent of this lost mass (mass defect) when divided by the number A of the nucleus is referred to as the binding energy per nucleon for the given nucleus. If a given nucleus has a total binding energy (E_b), its mass $_Z^A M$ can be correctly expressed through the relation.

$$_Z^A M = Zm_p + (A - Z)m_n - \frac{E_b}{c^2}$$

Von Weizsacker used the essential ideas of the liquid drop model to calculate the binding energy E_b in terms of number of terms such as volume energy, surface energy, Coulomb energy. In otherwords the binding energy of a nucleus is due to volume energy, surface energy and Coulomb energy according to liquid drop model.

1. Volume energy

We know that the binding energy per nucleon has an approximately constant value over a wide range of mass numbers. Hence as a first approximation, we can regard the binding energy of a nucleus to be proportional to the total number of nucleons (A) in it. Therefore it is proportional to nuclear volume, hence called volume energy (E_v).

$$\therefore \qquad E_v \propto A$$

or $\qquad\qquad E_v = a_v A \qquad\qquad\qquad\qquad ..... (1)$

The constant of proportionality a_v is the binding energy per nucleon.

2. Surface energy

In writing the above expression for volume energy, we have implicitly assumed that all the nucleons are attracted by all the other nucleons. However, this is not quite true. The surface nucleons have fewer nearer neighbours than the nucleons which are deep within the nuclear volume. This gives rise to a slight decrease in the total binding energy of the nucleus. This decrease is proportional to the number of nucleons on the surface. Hence it is proportional to the surface area of the nucleus. If r is the radius of the nucleus then we know that

$$r = r_0 \ A^{\frac{1}{3}}$$

Since the surface area is proportional to r^2, it must be proportional to $A^{\frac{2}{3}}$. Thus the contribution of the surface effect to the binding energy of the nucleus can be written as

$$E_s \propto A^{\frac{2}{3}}$$

or
$$E_s = - a_s \ A^{\frac{2}{3}}$$ ..... (2)

where a_s is an empirical constant. –ve sign shows that the binding energy gets reduced. This effect is similar to that in a liquid drop where the surface nucleons are less tightly bound and therefore evaporate out more easily from the liquid.

Coulomb energy

The electrostatic repulsion between each pair of protons in a nucleus also contributes toward decreasing its binding energy. The Coulomb energy (E_c) of a nucleus is the work that must be done to bring together Z protons from infinity into a spherical aggregate the size of the nucleus. The potential energy of a pair of protons r apart is equal to

$$V = - \frac{e^2}{4\pi\varepsilon_0 r}$$

Since there are $\frac{Z(Z-1)}{2}$ pairs of protons

$$E_c = \frac{Z(Z-1)}{2} V \ = - \frac{Z(Z-1)}{8\pi\varepsilon_0} \left(\frac{1}{r} \right)_{av}$$

where $\left(\dfrac{1}{r}\right)_{av}$ is the value of $\dfrac{1}{r}$ averaged over all proton pairs. If the protons are uniformly distributed throughout a nucleus of radius R, $\left(\dfrac{1}{r}\right)_{av}$ is proportional to $\dfrac{1}{R}$ and hence to $\dfrac{1}{A^{\frac{1}{3}}}$.

$$\therefore \qquad E_c = -a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

The Coulomb energy is negative because it arises from an effect that opposes nuclear stability.

Thus the total binding energy is the sum of its volume, surface and Coulomb energies.

$$E_b = E_v + E_s + E_c$$

$$E_b = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

$\therefore$ The binding energy per nucleon is

$$\frac{E_b}{A} = a_v - a_s A^{\frac{1}{3}} - a_c Z(Z-1)A^{-\frac{4}{3}}.$$

A graph is drawn between $\dfrac{E_b}{A}$ and A by appropriately choosing the coefficients so as to resemble as closely as possible to the empirical binding energy curve. This shows that liquid drop model has some validity.

Corrections to the formula

The binding energy formula derived in the last section is modified by taking two more factors which donot come under liquid drop model. This makes sense in terms of a model that provides for nuclear energy levels (shell model). The factors which contribute to binding energy are asymmetry energy and pairing energy.

Asymmetry energy

We found that among different isotopes of nuclei, the ones having equal number of protons and neutrons will be most stable and will therefore possess maximum binding energy. Thus we can say that an excess of neutrons over protons leads to a

decrease in binding energy and causes instability. This is called asymmetry energy contribution to binding energy.

We know that like energy levels in atoms, nuclear energy levels also exist. Nucleons which have spin $\frac{1}{2}\hbar$ obey Paulis exclusion principle. Hence each nuclear energy level can contain two neutrons of opposite spins and two protons of opposite spins. Energy levels in nuclei are filled in sequence to achieve configuration of minimum energy and maximum stability. When nucleus contains equal number of neutrons and protons all lower energy levels are filled hence it is stable. When the neutrons in a nucleus outnumber the protons, the higher energy levels have to be filled (see figure below).
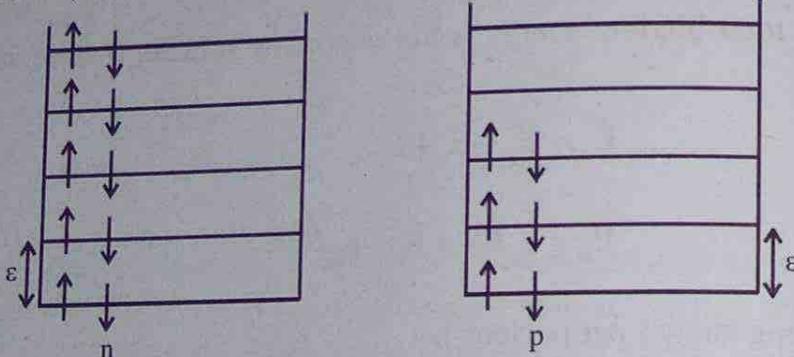

Figure 1.4

Let us suppose that the uppermost neutron and proton energy levels have the same spacing ε. Inorder to produce a neutron excess say $N - Z$ without changing
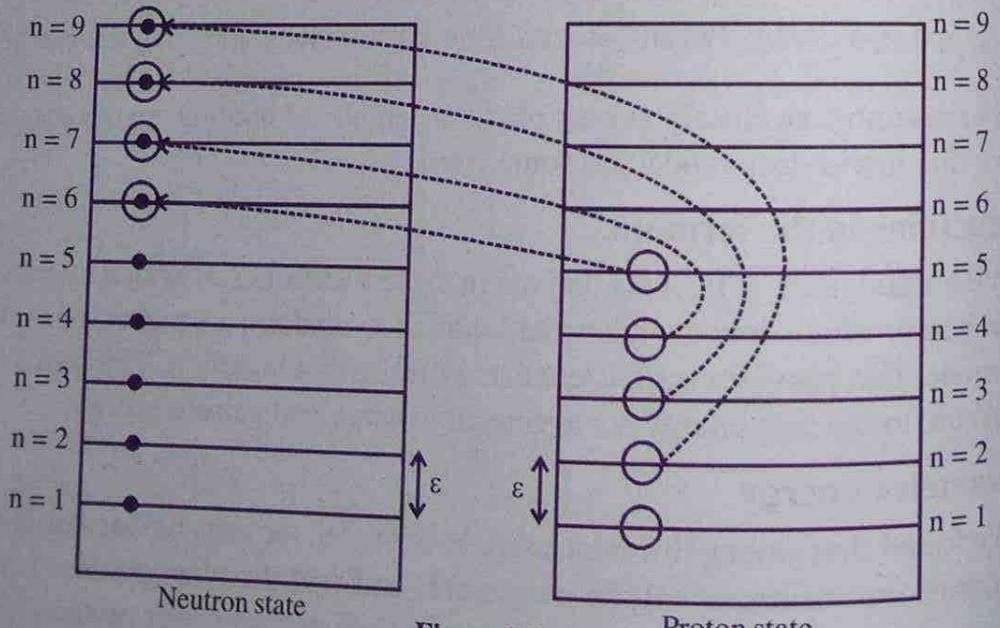

Figure 1.5

A, $\frac{1}{2}(N-Z)$ neutrons would have to replace protons in an original nucleus in which

$N = Z$.

[This is because when we add one neutron, one proton has be removed to keep A constant. i.e., for each neutron replacement two neutrons are increased. So to have

$N-Z$ excess neutrons only $\frac{N-Z}{2}$ neutrons will have to replace protons in the

original nucleus in which $N = Z$]

Now will calculate the total energy change when $\frac{N-Z}{2}$ neutrons are added by

replacing protons. There are ways of doing this. If we remove a proton from level 5 and add a neutron to level 6 the work required to be done for this is ε.

Now remove the proton from level 4 and add a neutron to level 7, the work required to be done for this is $3\varepsilon \, (E_7 - E_4 = 3\varepsilon)$.

To add 2 neutrons total work done $= \varepsilon + 3\varepsilon = 4\varepsilon$

Now remove the proton from levels 3 and add a neutron to level 8, the work required to be done for this is $5\varepsilon (E_8 - E_3 = 5\varepsilon)$.

$\therefore$ To add 3 neutrons total work done $= \varepsilon + 3\varepsilon + 5\varepsilon = 9\varepsilon$

Now remove the proton form level 2 and add a neutron to level 9, the work required to be done for this is $7\varepsilon \, (E_9 - E_2 = 7\varepsilon)$

$\therefore$ To add 4 neutrons total work done $= \varepsilon + 3\varepsilon + 5\varepsilon + 7\varepsilon = 16\varepsilon$

Now we summarise

Work done to add one neutron $= \varepsilon$

Work done to add two neutrons $= 4\varepsilon = 2^2 \varepsilon$

Work done to add three neutrons $= 9\varepsilon = 3^2 \varepsilon$

Work done to add four neutrons $= 16\varepsilon = 4^2 \varepsilon$

In general, work done to add $\left(\dfrac{N-Z}{2}\right)$ neutrons

$$= \left(\frac{N-Z}{2}\right)^2 \varepsilon$$

∴ The total energy change $= \left(\dfrac{N-Z}{2}\right)^2 \varepsilon$.

This energy change is called asymmetry energy which contributes to the binding energy (E_a)

i.e., $E_a = -\left(\dfrac{N-Z}{2}\right)^2 \varepsilon$

Negative sign shows that this contribution decreases the binding energy

Using $N = A - Z$

$E_a = -\left(\dfrac{A-2Z}{2}\right)^2 \varepsilon$

But the spacing of energy level decreases with A, according to $\varepsilon \propto \dfrac{1}{A}$.

Thus the asymmetry energy contribution can be written as

$E_a = -a_a \dfrac{(A-2Z)^2}{A}$

Pairing energy

This correction term arises from the following observations of nuclides. We find that

(i) All nuclides having an even number of protons and even number of neutrons are generally the most stable of all.

(ii) All nuclides having an odd number of protons and an odd number of neutrons are the least stable of all.

(iii) Nuclides belonging to even-odd (even Z, odd N) or odd-even (odd Z, even N) category have an intermediate level of stability. Of the two categories of these even - odd nuclides are relatively more stable than odd-even nuclides.

Depending upon the pairing, stability of nuclides vary so also binding energy. For even-even nuclides all the protons as well as neutrons are paired off and this makes them the most stable. For example nuclei such as $_2^4\text{He}$, $_6^{12}\text{C}$, $_8^{16}\text{O}$ are most stable and appears as peak on the empirical curve of binding energy per nucleon. For odd-odd nuclides there is at least one unpaired proton and one unpaired neutron and

this makes them least stable and have relatively low binding energies. For the even - odd or odd- even case we have one unpaired proton or one unpaired neutron and this makes them an intermediate stability. To take account of this pairing effect, pairing energy term is is produced. The pairing energy E_p is positive for even - even nuclei, for odd-even and even-odd nuclei and negative for odd-odd nuclei. On the basis of more detailed analysis, it is seen that, the pairing energy varies with A as $A^{-\frac{3}{4}}$.

i.e., $E_p = a_p A^{-\frac{3}{4}}$

where a_p is called the pairing energy constant.

Hence we have

$E_p = a_p A^{-\frac{3}{4}}$ for even-even

$E_p = -a_p A^{-\frac{3}{4}}$ for odd-odd.

$E_p = 0$ for even-odd, and odd-even.

Now taking all contributions to binding energy, we get

$$E_b = E_v + E_s + E_c + E_a + E_p$$

i.e.,

$$E_b = a_v A - a_s A^{\frac{2}{3}} - a_c Z(Z-1)A^{-\frac{1}{3}} - a_a \frac{(A-2Z)^2}{A} + E_p$$

This is called the semiempirical binding energy formula obtained by Von. Weizscaker in 1935. This is also called as Weizsacker formula. From this semi empirical mass formula can be written down.

The mass of nucleus is given by

$$_Z^A M = Zm_p + (A-Z)m_n - \frac{E_b}{c^2}$$

Substituting for E_b, we get

$$_Z^A M = Zm_p + (A-Z)m_n - \frac{1}{c^2}\left[a_v A - a_s A^{\frac{2}{3}} - a_c Z(Z-1)A^{-\frac{1}{3}} - a_a \frac{(A-2Z)^2}{A} + E_p \right]$$

This is called the semiempirical mass formula. Experimental results give the following values of the constants occurring in the semi empirical mass formula.

$a_v = 15.753$ MeV

$$a_s = 17.80 \text{ MeV}$$

$$a_c = 0.713 \text{ MeV}$$

$$a_a = 23.6925 \text{ MeV}$$

$$a_p = 33.6 \text{ MeV}.$$

However it must be kept in mind that no single unique set of values satisfies the equations for all known nuclides. Other sets of values of the constants may be found.

The semi empirical formula can account for the stability of nuclei against α and β decay and also it estimates the masses of wide range of nuclides. This shows that the liquid drop model is a good approximate model.

But liquid model fails to explain high stability of nuclei with magic numbers. This model also doesnot explain the measured spins and magnetic moments of the nuclei.

Example 8

Isobars are nuclides that have the same mass number A. Derive a formula for the atomic number of the most stable isobar of a given A and use it to find the most a stable isobar of A = 25, $a_c = 0.595$ and $a_a = 19$.

Solution

We have

$$E_b = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_c \frac{(A-2Z)^2}{A} + E_p$$

For stability E_b must be maximum.

i.e., $$\frac{dE_b}{dZ} = 0.$$

Different outing the above equation with respect to Z, we get

$$\frac{dE_b}{dZ} = -a_c A^{-1/3}(2Z-1) + \frac{4a_c}{A}(A-2Z) = 0$$

or $$\frac{4a_c}{A}(A-2Z) = a_c A^{-1/3}(2Z-1)$$

$$4a_c A - 8a_a Z = a_c A^{2/3} 2Z - a_c A^{2/3}$$

$$4a_a A + a_c A^{2/3} = Z(2a_c A^{2/3} + 8a_a)$$

$$\therefore \quad Z = \frac{4a_a A + a_c A^{2/3}}{2a_c A^{2/3} + 8a_a}$$

For A = 25

$$Z = \frac{4 \times 19 \times 25 + 0.595 \times (25)^{2/3}}{2 \times 0.595 \times (25)^{2/3} + 8 \times 19}$$

$$Z = \frac{1900 + 5.087}{10.174 + 152} = \frac{1905.087}{162.174}$$

$$Z = 11.747 \simeq 12$$

so the nuclide is $^{25}_{12}\text{Mg}$ which is obviously a stable isobar.

Shell model

The nuclear shell model is one of the most important and useful models of nucleus structure. According to this model the nucleons (protons and neutrons) have been interpreted as forming closed shells of neutrons and protons in analogy with the filling of electrons shells in atoms and the neutron and proton shells appear to be independent of each other. This model is based on the assumption that there exists a common nuclear potential within the nucleus and all the nucleons move nearly independent within this common nuclear potential.

The emergence of shell model

According to atom model the electrons in an atom are occupying positions in shells designated by the various quantum numbers. Certain important aspects of atoms behaviour is determined by the number of electrons filled in the outer most shell. For example, atoms with 2, 10, 18, 36, 54 and 86 electrons have all their electron shells completely filled. Such electron structures have high binding energies and exceptionally stable, which accounts for the chemical inertness of the rare gases.

The same kind of effect is observed in nuclei. Nuclei that have 2, 8, 20, 28, 50, 82 and 126 neutrons or protons are stable. Stability is related to high binding energy and also to high natural abundance. This is experimentally found to be true for all numbers 2, 8, 20, 28, 50, 82 and 126. These numbers are commonly referred to as magic numbers. The above stated similarity between atoms and nuclei motivated scientist to propose shell model to nuclear structure.

Another interesting property exhibited by nuclei with magic N or Z is that with regard to electric quadrupole moments. The magnitude of the quadrupole moment is a measure of the deviation of a nucleus from spherical shape. i.e., A spherical nucleus has no quadrupole moment, while one shaped like football has positive moment and one shaped like a pumpkin has a negative moment. Nuclei of magic N and Z are found to have zero quadrupole moment and hence are spherical, while the other nuclei are distorted in shape.

1. Hydrogen $^{4}_{2}H$ (Z = 2, N = 2)

2. Oxygen $^{16}_{8}O$ (Z = 8, N = 8)

3. Sulphur $^{36}_{16}S$ (Z = 16, N = 20)

4. Calcium $^{40}_{20}C_a$ (Z = 20, N = 20)

5. Molybdenum $^{93}_{42}M_o$ (Z = 42, N = 50)

6. Lead $^{208}_{82}pb$ (Z = 82, N = 126)

Nuclei for which both Z and N are magic numbers are called doubly magic $^{4}_{2}H$, $^{16}_{8}O$, $^{40}_{20}C_a$ are doubly magic and are particularly tightly bound.

Nuclear energy levels from the shell model

According to shell model each nucleon is moving independently in a nuclear potential. But the exact form of the potential energy function is not known unlike the case of an atom. Hence a suitable function has to be assumed. A reasonable guess is a square well potential with rounded corners. Having thought of this nuclear potential, we can solve the Schrodinger equation for the motion of this nucleon. The results obtained are similar to those for electrons orbiting around the force field of the nucleus. We find that there exists a set of discreate allowed energy levels where each of these energy levels has a specific value of the principle quantum number n associated with it. We also find that we must also associate an orbital quantum number l with a given nucleon. This quantum number tells us about the discrete set of values of the orbital angular momentum. In essence the energy of a given nucleon is given by the quantum numbers n and l. Further like in the case of electrons we have

to think of the nucleons having spin quantum numbers $\left(\pm \dfrac{1}{2} \hbar \right)$. This implies that both proton and the neutron are fermions and hence obey Paulis exclusion principle.

The similarity between these pictures of the electrons and nucleons implies that

(i)   An energy state with a given value of l can accommodate a total of $2(2l+1)$ nucleons. For each value of l there are $2l+1$ sub states. The exclusion principle limits the number of neutrons or protons occupying a level to $2(2l+1)$.

(ii)   We can designate the nuclear energy states as s, p, d, g, f for orbital angular momentum quantum number $l = 0, 1, 2, 3, 4$.

Explanation of magic numbers

We found that a nuclear energy level with a given value of l can accommodate $(2l+1)$ nucleons. So the nuclear closed shells can contain either 2 or $2+6=8$, or $2+6+10=18$, or $2+6+10+2=20$, or $2+6+10+2+14=34$ and so on. The magic numbers 2, 8, 20 are thus understood easily. The other magic numbers 28, 50, 82, 126 etc. however, cannot be explained on this basis. To circumvent this difficulty a new concept of strong interaction between the orbital angular momentum $\vec{L}$ and the spin angular momentum $\vec{S}$. There are two ways of interaction between $\vec{L}$ and $\vec{S}$. One is called LS coupling and the other is called J J coupling. The shell model assumes that LS coupling holds only for lightest nuclei and JJ coupling holds for heavier nuclear. In the case of LS coupling the spin angular momentum S_i of the particles (protons and neutrons separately) are coupled together into a total spin angular momentum S. The orbital angular momentum L_i are separately coupled together into a total orbital angular momentum $\vec{L}$. Then $\vec{L}$ and
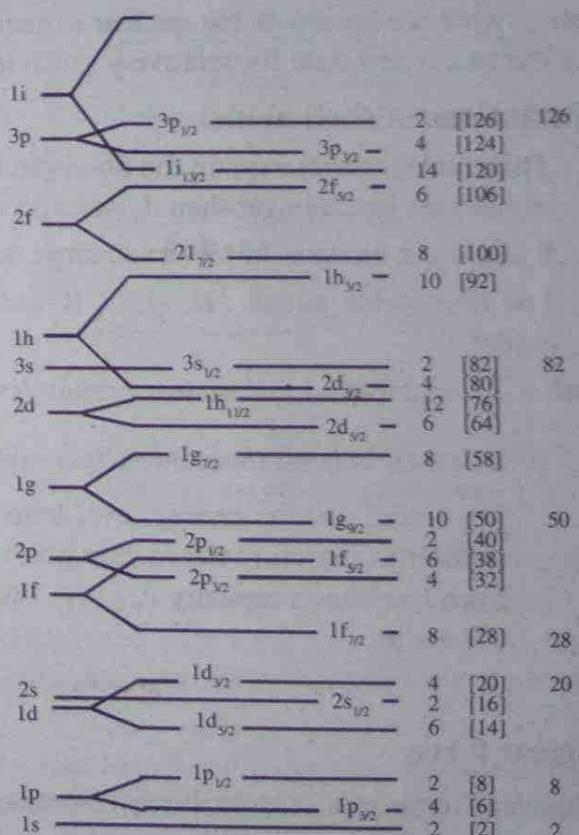

Figure 1.6: Energy levels of a nucleon in the potential well according to shell model

$\bar{S}$ are coupled to form a total angular momentum $\bar{J}$ with magnitude $\sqrt{J(J+1)}\,\hbar$. In the case of JJ coupling the S_i and L_i of each particle are first coupled to form a j_i for that particle of magnitude $\sqrt{j(j+1)}\,\hbar$. The various j_i are coupled to form the total angular momentum J. The modified picture then helps us to understand the whole magic numbers. See the nuclear energy level diagram (figure 1.6).

The merits of the shell model

The shell model has proved useful and successful in explaining and accounting for the observed

(i)   Angular momentum.

(ii) Magnetic moment.

(iii) Electric quadrupole moment.

Besides these, the shell model also helps us to understand the observed distribution of what are known as the nuclear isomers. Nuclear isomers are nuclei that can exist in an excited state for relatively much longer times (half life $\geq$ 1s).

Limitations of shell model

1. This model cannot explain the observed first excited states in even-even nuclei at energies much lower than those expected from single particle excitation.

2. It could not explain the observed large quadrupole moments of odd nuclei.

3. The four stable nuclei $^{2}_{1}H$, $^{6}_{3}L_i$, $^{10}_{5}B$ and $^{14}_{7}N$ donot fit in the scheme of this model.

Note :   To understand the nuclear energy level (figure above) note the following.

The total angular momentum has values $j = l + \dfrac{1}{2}$ and $j = l - \dfrac{1}{2}$. Empirically it is found that the energy level with higher value of j lies below that with smaller j. Therefore the former gives a more tightly bound nucleonic state. Each level has a capacity $(2j+1)$. The ordering of l for all the unsplit levels of l are as

<div align="center">spdsfpgdshfpig</div>

Nuclear force

Nuclear forces are responsible for keeping nucleons together. Repulsive force between protons inside nucleons would tear nucleons apart. Since nucleus is stable, there must be a force which keeps nucleons together is called nuclear force.

The only way to learn about nuclear force is from experiments. The scattering

experiments of neutrons and protons and other varieties of experiments, the following characteristics of nuclear force have been noted.

Properties of nuclear force

1. The nuclear force is the strongest of the four known forces, hence called strong force. For two adjacent protons in a nucleus, the nuclear interaction is 10 - 100 times stronger than the electromagnetic interaction.

2. The nuclear force is short range.

   The distance over which the nuclear force acts is limited to about 10^{-15}m. This comes from the fact that central density of nuclear matter is constant. As we add nucleons to be nucleus, each added nucleon feels a force only from its neighbours and not from all other nucleons in the nucleons. In this respect, a nucleus behaves somewhat like a crystal, in which each atom interacts primarily with its neighbours and addition of atoms make the crystal larger but don't change its density. Another piece of evidence comes from the relation between nuclear binding energy and the separation distance of nucleons. It has been found that nuclear binding energy is a constant for separation distance is less than about 10^{-15}m and it is zero for separation distances greater than 10^{-15}m. (see figure).

   From this we can conclude that for short range strong force, the binding energy is proportional to A because binding energy per nucleon is constant. For a force with long range

   Figure 1.7: Dependence of nuclear binding energy on separation distance

   (gravitational and electrostatic forces having infinite range) the binding energy is roughly proportional to the square of the number of interacting particles $(E_b \propto Z^2)$. This is because each of the Z protons in a nucleus feels the repulsion of the other $Z-1$ protons. Thus the total electrostatic energy of the nucleus is proportional to $Z(Z-1)$. For large Z it is Z^2.

3. The nuclear force between any two nucleons does not depend on whether the nucleons are protons or neutrons. The n-p nuclear force is the same as the n-n nuclear force which is in turn same as the p-p nuclear force.

4. The nuclear force is not completely central force. It depends on the orientation of the spins relative to the line joining the two nucleons. This property has been

deduced by noting that even in the simplest nucleus (deuterium), the orbital angular momentum of the two nucleons relative to the centre of mass is not constant whereas for central forces angular momentum is conserved.

A successful model for the origin of this short range force was put for ward by Japanese physicist Yukawa. According to this model some particles are responsible for nuclear force. These particles are called pions. Pions may be charge π^+, π^- or neutral π^0. According to this model every nucleon continuously emits and reabsorbs pions. The transfer of momentum associated with the shift of pion is equivalent to the action of a force. Thus nuclear force is the result of the exchange of pions between the nucleons. That is nuclear force is an exchange force.

Two nucleons experience an attractive force at small distance because of the virtual exchange of pions between them. The situation, qualitatively, is analogous to two dogs grappling for control over a bone. One dog grabs the bone from the other and then, the second snatching it back again. The continual exchange of the bone results in each dog pulled towards the other.

Rough estimate of pion mass

According to uncertainty principle we have $\Delta E\, \Delta t \sim \hbar$

Let us assume that a pion travels between nucleons at a speed $v \sim c$, that the emission of a pion of mass m_π represents a temporary energy uncertainty $\Delta E \sim m_\pi c^2$. Since nuclear forces have a maximum range r of about 1.7fm and the time Δt needed for the pion to travel this distance is

$$\Delta t = \frac{r}{v} \sim \frac{r}{c}$$

$\therefore \qquad \Delta E\, \Delta t \sim \hbar$

or $\qquad m_\pi = \dfrac{\hbar}{rc} = \dfrac{1.05 \times 10^{-34}}{1.7 \times 10^{-15} \times 3 \times 10^8} = 2 \times 10^{-28}\, kg$

$$m_\pi = \frac{2 \times 10^{-28}\, m_e}{7.7 \times 10^{-31}} \simeq 220 m_e.$$

After 12 years of the prediction of pions, they were experimentally detected. The rest mass of the charged pions were found to be $273 m_e$ and that of neutral pions to be $264 m_e$. There were two reasons for the belated discovery of pion. One is that particles with kinetic energies of several hundred MeV energy was required to produce pions. This much energy producing accelerators were not available at that time. The second reason is the pions instability. The mean life time of the charged pion is only 2.6×10^{-8} s and that of the neutral pion is 8.4×10^{-17} s.

Radioactive decay

We found that for the lighter stable nuclei the proton and neutron numbers are roughly equal. However, for the heavy stable nuclei, the factor $Z(Z-1)$ in the Coulomb repulsion energy grows rapidly, so extra neutrons are required to supply the additional binding energy needed for stability. For this reason, all heavy stable nuclei have $N > Z$. Whenever the stability condition is upset they transform themselves into more stable nuclei by changing their Z and N through alpha decay or beta decay. Nuclei are unstable in excited states and they undergo gamma decay to achieve stability.
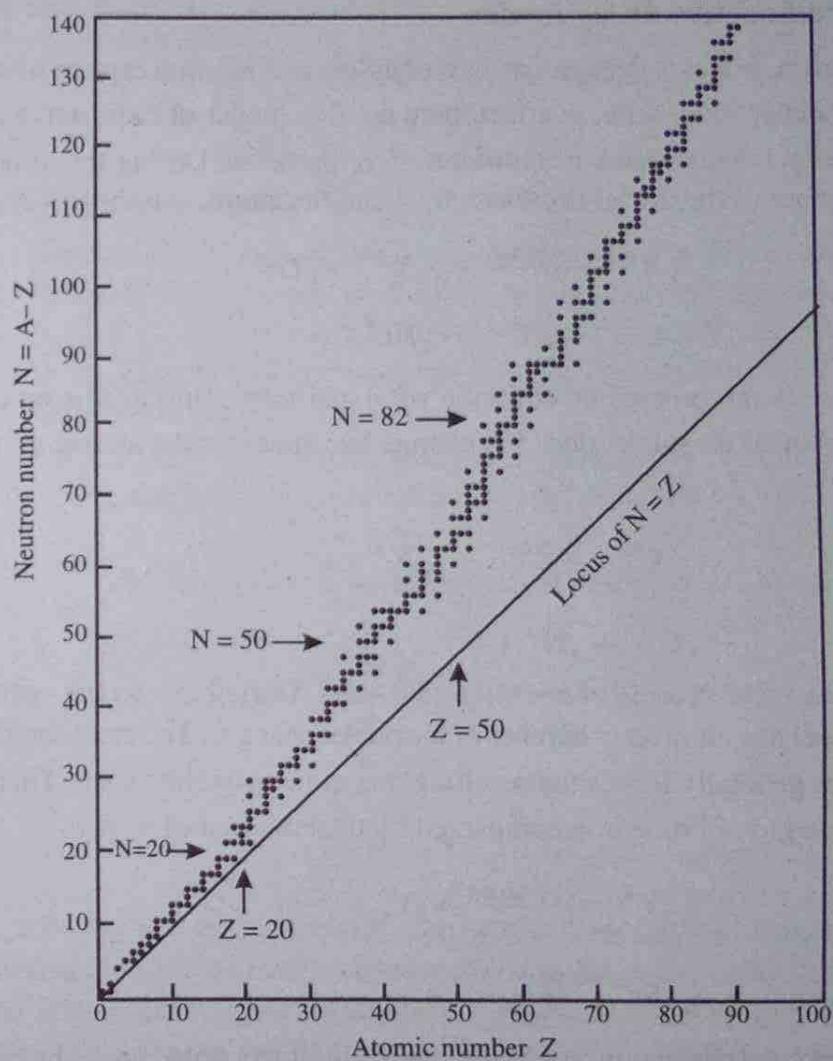
**Figure 1.8: N-Z plot of stable nuclides**
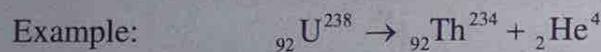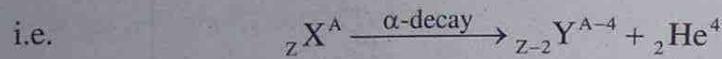
Which type of decay is occurring can be easily understood from the proton number versus neutron number plot. A graph is plotted between proton number (Z) on the horizontal axis and neutron number on the vertical axis by taking all known stable nuclides. We get a graph shown above. The dotted points represent the stable nuclides called stability curve.

Nuclei to the right of the stability line where the protons dominate over neutrons, they undergo β_+ decay. Nuclei to the right of the stability line where the neutrons dominate over protons, they undergo β_- decay. When the proton number is greater than $82(Z > 82)$, they undergo alpha decay.
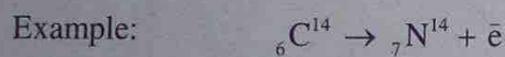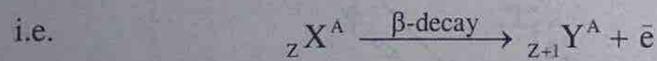
Different radioactive decay modes

Apart from α, β and γ decays, positron emission and electron capture were added to the list of decay modes. i.e. in effect there are five modes of radioactive decay.
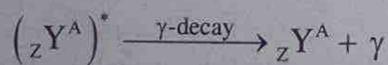
1. Alpha decay is the process of emission of α particles. During this process the mass number of the nuclei decreases by 4 and the atomic number by 2.

   i.e. $$_{Z}X^{A} \xrightarrow{\quad \alpha\text{-decay} \quad} {}_{Z-2}Y^{A-4} + {}_{2}He^{4}$$

   Example: $$_{92}U^{238} \rightarrow {}_{92}Th^{234} + {}_{2}He^{4}$$

2. Beta decay is the process of emission of β-particles. During this process the mass number of the nuclei does not change but increases the atomic number by one.

   i.e. $$_{Z}X^{A} \xrightarrow{\quad \beta\text{-decay} \quad} {}_{Z+1}Y^{A} + \bar{e}$$

   Example: $$_{6}C^{14} \rightarrow {}_{7}N^{14} + \bar{e}$$
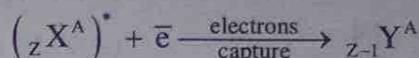
3. Gama decay is the process of emission of γ-rays. During this process neither the mass number nor the atomic number of the nuclei changes. The emission of α and β-particles generally leaves the resulting nuclei in an excited state. Then return to the stable ground state is accompanied by the emission of γ-rays.

   $$\left(_{Z}Y^{A}\right)^{*} \xrightarrow{\quad \gamma\text{-decay} \quad} {}_{Z}Y^{A} + \gamma$$

   $$\left(_{38}Sr^{87}\right)^{*} \rightarrow {}_{38}Sr^{87} + \gamma$$

4. When the excited nuclei is proton rich it often returns to a stable nuclei by capturing an orbital (K-shell) electron. This process is called electrons capture or K-capture.
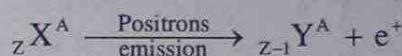
The vacancy caused in the K.shell is filled by transition of electrons from higher orbits with subsequent emission of characteristic x-rays.

$$\left(_Z X^A\right)^{*} + \bar{e} \xrightarrow[\text{capture}]{\text{electrons}} {}_{Z-1}Y^A$$

It is regarded as another form of β-decay since during this process the mass number remains the same and the atomic number decreases by one.

Example: $\qquad _{29}Cu^{64} + \bar{e} \rightarrow {}_{28}Ni^{64}$

5. Positron emission is the process of emission positrons. During this process the mass number remains unchanged but atomic number decreases by one.

$$_Z X^A \xrightarrow[\text{emission}]{\text{Positrons}} {}_{Z-1}Y^A + e^+$$

Example: $\qquad _{29}Cu^{64} \rightarrow {}_{28}Ni^{64} + e^+$

Law of radioactive decay

The disintegration of radioactive nucleus was studied by Rutherford and Soddy in 1902. Whatever be the nature of decay, it is essentially a statistical process. In a given sample of a radioactive substance, the various identical nuclei take different times to decay. There is absolutely no way of predicting the moment of decay of any given nucleus in the sample.

In general, the statistical character of the decay process can be expressed as follows. If a radioactive sample contains N nuclei at a given instant the ratio of the rate of decay $\left(-\dfrac{dN}{dt}\right)$ to the number of nuclei present at that instant is a constant

Thus $\qquad \dfrac{-\dfrac{dN}{dt}}{N} = \lambda \text{ (constant)}$

or $\qquad \dfrac{dN}{dt} = -\lambda N \qquad\qquad ..... (15)$

The constant λ is called the decay constant or disintegration constant. Its value is characteristics of the radioactive substance and signifies the decay probability of the sample. The negative sign indicates that N is decreasing with time. Equation (15) expresses the law of radioactive decay. The following interesting characteristics of the radioactive substance can be obtained from this law.

(i) Exponential decay

Equation (15) can be written in the form

$$\frac{dN}{N} = -\lambda \, dt$$

Integrating both sides, we get

$$\ln N = -\lambda t + C \qquad\qquad .....(16)$$

at $t = 0$, $N = N_0$

$$\therefore \qquad \ln N_0 = -\lambda \times 0 + C$$

or $\qquad\qquad\quad C = \ln N_0$

$\therefore \qquad$ Eq (16) becomes

$$\ln N = -\lambda t + \ln N_0$$

or $\qquad \ln N - \ln N_0 = -\lambda t$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

i.e. $\qquad\qquad\qquad \dfrac{N}{N_0} = e^{-\lambda t}$

$\therefore \qquad\qquad\qquad N = N_0 e^{-\lambda t} \qquad\qquad .....(17)$

Half Life

The time interval during which half of the atoms of the given radioactive sample decay is called half life. It is denoted by T

i.e., when $t = T$, $N = \dfrac{N_0}{2}$

using $\qquad N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

$$\frac{1}{2} = e^{-\lambda t}$$

$$2 = e^{\lambda T}$$

i.e., $\qquad \ln 2 = \lambda T$

$$2.303 \times \log 2 = \lambda T$$

$$2.303 \times 0.3010 = \lambda T$$

$$0.693 = \lambda T$$

Hence $$T = \frac{0.693}{\lambda} \qquad .....(18)$$

Thus half life of a radioactive substance is a characteristic property and is inversely proportional to the decay constant. It cannot be changed by any chemical or physical means. For example half life of radium is 1620 years, that of $_{92}U^{238}$ is 4.51×10^9 years and $_{84}Po^{214}$ is 10^{-6} s. In numerical problems, instead of specifying decay constant, usually half life of a nucleus is specified. The equation (18) is a convenient expression for relating half life to the decay constant. Note that after one half life, $N_0/2$ radioactive nuclei remain; after two half lives $N_0/4$ radioactive nuclei are left; after three half lives, $N_0/8$ are left; and so on. In general, the number of nuclei remaining after n half lives is $N_0/2^n$. The decay of a radioactive nucleus is a statistical process. If we take a radioactive sample of 1mg with a half life of 1h, about 50% of the 1 mg sample will decay in 1h ; during the second hour probability of decay is still 50%, for each remaining nucleus. The total probability that a given nucleus did not decay is $0.5 \times 0.5 = 0.25$ or 25%. The probability of decay is 75%, which is a fraction of the original nucleus expected to be disintegrated in 2 h.


Figure 1.9

Note : After n half lives $$N = N_0 \left(\frac{1}{2}\right)^n$$

we have $$N = N_0 e^{-\lambda t}$$

If $$\lambda = \frac{1}{t}, N = N_0 e^{-1}$$

or $$N = \frac{N_0}{e} = \frac{N_0}{2.718} = 0.368 N_0 = 36.8\% N_0$$
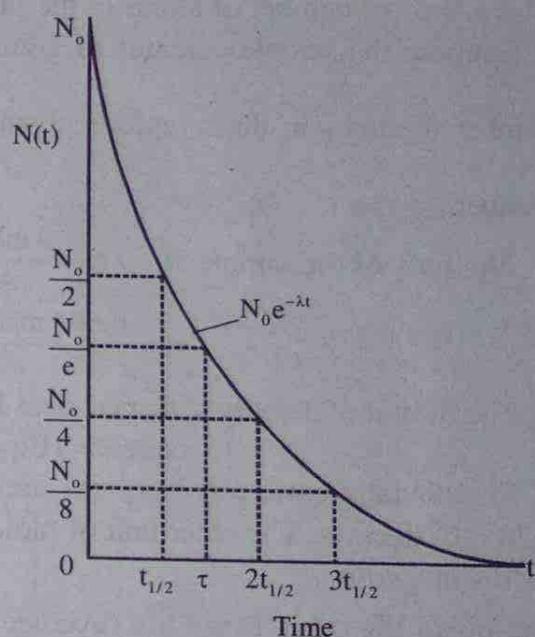
Thus decay constant may be defined as the inverse of the time during which the number of the atoms in the radioactive sample is reduced to 36.8% of their original value.

Activity of the given sample

The number of disintegrations per second of the radioactive sample is called activity. It is denoted by R.

i.e.,
$$R = \frac{dN}{dt} = \lambda N$$

where N is the number of atoms of the sample

Suppose the sample contains m grams of the radio active element $_zX^A$. The number of atoms in the sample is given by $N = \dfrac{mNa}{A}$ where Na is the Avagadro number,

∴   Activity of the sample $R = \lambda N = \dfrac{\lambda mNa}{A}$

$$R = \frac{0.693}{T} \frac{mNa}{A}$$

The SI unit of activity is named after Becquerel.
$$1 \text{becquerel} = 1 Bq = 1 \text{decay/s}$$

Traditionally curie (ci) has been used as the unit of activity, 1curie = 1Ci = 3.70×10^{10} decay/s. The other unit of radioactivity is rutherford (Rd), 1rutherford = 10^6 disintegration /s.

The mean life time: Mean life (average life) τ is defined as the average time the nucleus survives before it decays. Mean life can be determined by calculating total life time of all the nuclei initially present (N_0) and dividing it by total number of nuclei. Let the number of nuclei decaying in time intervals t and t+dt be dN, and the life time of these nuclei be t. Then the total life time of these nuclei is t dN.

The total life time of all the nuclei $\displaystyle\int_{N=N_0}^{N} t dN$

From equation (3), $dN = N_0 \lambda e^{-\lambda t} dt$
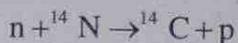
Now recognising the fact that N = 0 for $t \to \infty$ and $N = N_0$ for $t \to 0$, total life time of all the nuclei is

$$= \int_{N=N_0}^{N} t dN = \int_0^\infty t N_0 \lambda e^{-\lambda t} dt = N_0 \lambda \int_0^\infty t e^{-\lambda t} dt = \frac{N_0}{\lambda}$$

(we can carry out integration by parts to get the result). Thus the mean can be obtained by dividing the total life of all the nuclei to decay by the total number of nuclei. Thus,

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$$

Radioactive carbon dating : Radioactive ^{14}C is produced in our atmosphere by the bombardment of ^{14}N by cosmic rays.

$$n + {}^{14}N \rightarrow {}^{14}C + p$$

The ratio of ^{14}C to ^{12}C in the carbon dioxide molecules of our atmosphere has a constant value of approximately 1.3×10^{-19}. All the living organisms exchange carbon dioxide from atmosphere; hence a constant ratio of ^{14}C to ^{12}C is maintained in them. When an organism dies, it does not absorb ^{14}C from atmosphere, hence the ratio of ^{14}C to ^{12}C decreases as a result of beta decay of ^{14}C, which has a half life of 5730 year. It is therefore possible to measure the

Plants and animals use of breathe CO_2

When an organism dies, the ratio of $^{14}C/^{12}C$ decreases

Figure 1.10

age of a material by measuring its activity caused by radioactive ^{14}C.

Example 9

The half life of Radon is 3.8 days. Calculate how much of 15 milligram of Radon will remain after 38 days.

Solution

T = 3.8 days, t = 38 days

Number of half lives in 38 days is

$$n = \frac{38}{3.8} = 10$$

$$N = N_0 \left(\frac{1}{2}\right)^n$$

$$= 15 \times \left(\frac{1}{2}\right)^{10} = 0.014 \, mg$$
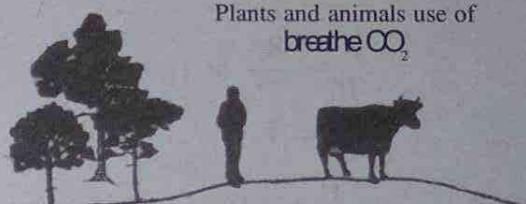
Example 10

Find the decay constant of a sample of the radio active sample whose activity becomes $\frac{1}{16}$ th in 10 years

Solution

Since activity is proportional to the number of atoms in the sample. Therefore activity after n half lives is given by

$$R = \frac{R_o}{2^n}, \text{ where } R_o \text{ is the activity in the beginning}$$

Here $R = \dfrac{R_o}{16}$

$\therefore$ $\dfrac{R_o}{16} = \dfrac{R_o}{2^n}$ or $16 = 2^n$

i.e., $n = 4$

We have $n = \dfrac{t}{T}$, $t = 10$ years (given)

$$4 = \frac{10}{T}$$

$$T = 2.5 \text{ years}$$

$\therefore$ decay constant $\lambda = \dfrac{0.693}{2.5} = 0.277 / \text{year}$

Example 11

The half life of a radio active sample is 4 days. What fraction of 1 gram sample will remain after 20 days.

Solution

Amount of substance left after n half lives is given by

$$M = \frac{M_n}{2^n}$$

$$M_o = 1g, \ n = \frac{20}{4} = 5$$

Hence $M = \dfrac{M_o}{2^n} = \dfrac{1}{2^5} = 0.03125 g$

Example 12

A radioactive substance has a half life period of 30 days. Calculate the time taken for 3/4 of original number of atoms to distintegrate.

Solution

$$T = 30 \text{days}$$

$$N = \frac{N_o}{4}$$

Number of half lives $n = \dfrac{t}{T} = \dfrac{t}{30}$

Using $\quad N = \dfrac{N_o}{2^n}$

$$\frac{N_0}{4} = \frac{N_0}{2^n}$$

$$4 = 2^n$$

$$n = 2$$

or $\qquad \dfrac{t}{30} = 2$

$$t = 60 \text{days}$$

Example 13

Determine the amount of $_{84}Po^{210}$ having activity equal to 5 millicurie. The half life of Po is 138 days.

Solution

Activity of the sample

$$R = \frac{0.693}{T}\left(\frac{mNa}{A}\right)$$

$$R = 5\text{mci} = 5 \times 10^{-3} = 18.5 \times 10^7 s^{-1}$$

$$(1\text{ci} = 3.7 \times 10^{10} s^{-1})$$

$m = ? \quad Na = 6.025 \times 10^{23}, A = 210$

$$T = 138 \times 24 \times 60 \times 60 s$$

$$m = \frac{T \times R \times A}{0.693 \times Na}$$

$$= \frac{138 \times 24 \times 60 \times 60 \times 18.5 \times 10^7 \times 210}{0.693 \times 6.025 \times 10^{23}}$$

$$= 1.1 \times 10^{-6} \, g$$

Example 14

The disintegration rate of a certain radioactive sample at any instant is 4750 disintegration per minute. Five minute later the rate of disintegration becomes 2700 disintegrations per minute. Calculate the half life of the sample.

Solution

$$\frac{dN_o}{dt} = -\lambda N$$

At $t = 0$ $\frac{dN_o}{dt} = -\lambda N_0$

After 5 minutes $\frac{dN}{dt} = -\lambda N$

Hence $\frac{N_0}{N} = \frac{(dN_0/dt)}{(dN/dt)} = \frac{4750}{2700} = 1.759$

Using $N = N_0 e^{-\lambda t}$

or $\lambda = \frac{1}{t} \times 2.303 \log\left(\frac{N_0}{N}\right) = \frac{1}{5} \times 2.303 \log(1.759)$

$$= 0.1130 / \text{minutes}$$

$$T = \frac{0.693}{\lambda} = \frac{0.693}{0.1130} = 6.13 / \text{minutes}$$

Example 15

A piece of burnt wood of mass 20g is found to have a C^{14} activity of 4 decays/s. How long has the tree that this wood belonged to been dead. Given $T_{1/2}$ of $C^{14} = 5730 \, \text{year}$.

Solution

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5730 \times 3.17 \times 10^7} = 3.83 \times 10^{-12} \, s^{-1}$$

∵ $1 \, \text{year} = 3.17 \times 10^7 \, s$

To find the number of C^{14} nuclei in 20g of burnt wood, we first calculate the number of C^{12} nuclei in 20g of carbon (burnt wood)

Thus $\quad N(C^{12}) = \dfrac{6.02 \times 10^{23}}{12} \times 20 \approx 10^{24}$

Now assuming that the ratio of C^{14} to C^{12} is 1.3×10^{-12}, the number of C^{14} nuclei in 20 g before decay is,

$$N_0(C^{14}) = (1.3 \times 10^{-12})(10^{24}) = 1.3 \times 10^{12}$$

We thus have for the initial activity of the sample

$$R_0 = N_0 \lambda = (1.3 \times 10^{12}) \times (3.83 \times 10^{-12})$$

$$= 4.979 \, \text{decay/s} \approx 5 \, \text{decay/s}$$

The age of the sample can now be calculated from the releation,

$$R = R_0 e^{-\lambda t}$$

or $\qquad e^{\lambda t} = \dfrac{R_0}{R}$

or $\qquad t = \dfrac{1}{\lambda} \ln\left(\dfrac{R_0}{R}\right)$

It is given that $R = 4$ decay/s and we have calculated $R_0 = 5 \, \text{decay/s}$.

Thus $\qquad t = \dfrac{1}{\lambda} \ln\left(\dfrac{5}{4}\right) = \dfrac{\ln(1.25)}{3.83 \times 10^{-12}} \, \text{s}$

$$= \dfrac{0.223}{3.83 \times 10^{-12}} \text{s} = 0.58 \times 10^{11} \text{s} = 1842 \, \text{year}$$

Example 16

Find the activity of 1mg of radon whose atomic mass is 222u. $T_{1/2} = 3.8$ days.

Solution

$$T_{1/2} = 3.8 \, \text{days} = 3.8 \times 86400 \text{s}$$

$$\therefore \qquad \lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{3.8 \times 86400} = 2.11 \times 10^{-6} \text{s}^{-1}$$

The number of atoms in 1mg of radon.

$$N = \frac{1 \times 10^{-3}}{222u} = \frac{10^{-3}}{222 \times 1.66 \times 10^{-27}} = 2.71 \times 10^{18}$$

$$\therefore \quad \text{Activity } R = \lambda N = 2.11 \times 10^{-6} \times 2.71 \times 10^{18}$$

$$= 5.72 \times 10^{12} \text{ decay s}^{-1}$$

$$= \frac{5.72 \times 10^{12}}{3.7 \times 10^{10}} \text{ Ci}$$

$$= 155 \text{ Ci}$$

Example 17

Find the probability that a particular nucleus of ^{38}Cl will undergo beta decay in any 1 second period. The half life of ^{38}Cl is 37.2 minutes.

Solution

$$T_{1/2} = 37.2 \text{ minutes} = 37.2 \times 60s$$

$$\therefore \quad \lambda = \frac{0.693}{37.2 \times 60} = 3.10 \times 10^{-4} s^{-1}$$

$$\therefore \quad \text{Probability} = 3.10 \times 10^{-4}$$

Example 18

The half life of the alpha emitter ^{210}Po is 138 days. What mass of polonium (210) is needed for a 10mCi source.

Solution

$$T_{1/2} = 138 \text{ days} = 138 \times 86400s$$

$$\therefore \quad \lambda = \frac{0.693}{138 \times 86400} = 5.812 \times 10^{-8} \text{ decay s}^{-1}$$

we have

$$R = \lambda N \quad R = 10m \text{ Ci (given)}$$

$$R = 10m \text{ Ci} = 10^{-2} \text{ Ci} = 10^{-2} \times 3.7 \times 10^{10} \text{ decay s}^{-1}$$

$$= 3.7 \times 10^{8} \text{ decay s}^{-1}$$

$$\therefore \quad N = \frac{R}{\lambda} = \frac{3.7 \times 10^{8}}{5.812 \times 10^{8}} = 6.366 \times 10^{15} \text{ atoms}$$

$$\therefore \quad \text{Mass of } 6.366 \times 10^{15} \text{ atoms} = \frac{6.366 \times 10^{15} \times 210}{6.02 \times 10^{23}}$$

$$= 22.2 \times 10^{-7} \, \text{g}$$

$$= 22.2 \times 10^{-10} \, \text{kg}$$

Conservation laws in radioactive decay

Our study of radioactive decays and nuclear reactions reveals that these processes occurring in nature are not arbitrarily but according to certain laws. The laws are called conservation laws. These laws put some limitations on the outcome of the processes. At the same time these laws give us important insight into the fundamental workings of nature. There are five conservation laws that can be applied to radioactive decays.

1. Conservation of energy

This states that energy before decay is equal to energy after decay. This enables us to calculate rest energies or kinetic energies of decay products. Consider a nucleus X decay into lighter nuclei X′ with the emission of one or more particles we call it collectively as x. Let m_X be the rest mass of the nucleus X, $m_{X'}$ be the rest mass of the nucleus after decay and m_x be the rest energy of the emitted particles.

According to the law of conservation of energy.

$$m_X c^2 = m_{X'} c^2 + m_x c^2 + \text{Energy released}$$

Energy released in the form of kinetic energy of decay products. This is usually called as the Q value.

Thus

$$m_X c^2 = m_{X'} c^2 + m_x c^2 + Q \qquad \qquad \dots (19)$$

or $$Q = [m_X - (m_{X'} + m_x)]c^2 \qquad \qquad \dots (20)$$

The decay is possible only Q values is positive. For this to be possible only if the rest energy of the nucleus X is greater than the total rest energy of the decay products X′ and x

i.e. $$m_X > m_{X'} + m_x$$

If the Q value appears as kinetic energy of the decay products

$$Q = K_{X'} + K_x \qquad \qquad \dots (21)$$

2. Conservation of linear momentum

The law states that the momentum of decaying nucleus before is equal to the momentum of the decay products. If the decaying nucleus is at rest initially it momentum is zero. After the decay the total momentum is $\vec{p}_{X'} + p_x$.

According to the law

Momentum before decay = Momentum after decay

i.e., $$0 = \vec{p}_{X'} + \vec{p}_x \qquad \qquad ..... (22)$$

Usually the emitted particles x are less massive than the residual nucleus X'. Thus the recoil momentum $p_{X'}$ yields a very small kinetic energy $K_{X'}$. This is because

$$K_{X'} = \frac{p_{X'}^2}{2m_{X'}}$$

and $$K_x = \frac{p_x^2}{2m_x}$$

∴ $$\frac{K_{X'}}{K_x} = \frac{p_{X'}^2}{2m_{X'}} \frac{2m_x}{p_x^2}$$

$$\frac{K_{X'}}{K_x} = \frac{m_x}{m_{X'}} \qquad \qquad (\because \quad p_{X'} = -p_x)$$
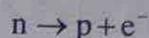
since $m_x < m_{X'}$, we get $K_{X'} < K_x$.

If there is only one emitted particle, solving equations 21 and 22 we get the values $K_{X'}$ and K_x. If two or more particles are emitted we get no unique solution. In this case a range of values from some minimum to some maximum is permitted for the decay products.

3. Conservation of angular momentum

This law states that the total spin angular momentum of the initial particle before the decay must be equal to the total angular momentum of all of the product particles after decay.

For example

$$n \rightarrow p + e^-$$

$$\frac{1}{2} \rightarrow \frac{1}{2} \pm \frac{1}{2}$$

Here total angular momentum is not conserved. Adding integer units of orbital angular momentum to the electron does not restore conservation.

4. Conservation of electric charge
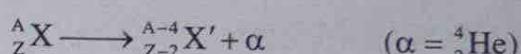
This law states the total charge before decay must be equal to the total charge of decay products.

5. Conservation of nucleon number

This law states that the total nucleon number A does not change in decay processes. In this protons can be transformed into neutrons or vice versa but $N + Z$ must be remain invariant.

Alpha decay

We found that in alpha decay an unstable nucleus decays into a lighter nucleus and an alpha particle. Thus we have

$$^A_Z X \longrightarrow ^{A-4}_{Z-2} X' + \alpha \qquad (\alpha = ^4_2 He)$$

This decay process releases energy in the form of kinetic energy of the decay products. The Q value of the process is

$$Q = \left[m_X - (m_{X'} + m_\alpha) \right] c^2$$

Here all m's are nuclear masses

and

$$Q = K_{X'} + K_\alpha \qquad \qquad \qquad ..... (24)$$

From the law of conservation of linear momentum, we have

$$O = \vec{p}_{X'} + \vec{p}_\alpha$$

$$O = m_{X'} v_{X'} + m_\alpha v_\alpha$$

or

$$m_{X'} v_{X'} = -m_\alpha v_\alpha$$

But

$$K_{X'} = \frac{1}{2} m_{X'} v_{X'}^2 = \frac{1}{2} m_{X'} \left(\frac{-m_\alpha v_\alpha}{m_{X'}} \right)^2$$

$$K_{X'} = \frac{1}{2} \frac{m_\alpha^2 v_\alpha^2}{m_{X'}}$$

or
$$m_X K_{X'} = \frac{1}{2} m_\alpha^2 v_\alpha^2 = \frac{1}{2} m_\alpha v_\alpha^2 \cdot m_\alpha$$

$$m_X K_{X'} = K_\alpha m_\alpha \qquad \left(\because K_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 \right)$$

Using $m_\alpha = 4$ and $m_X \simeq A - 4$, we get

$$(A - 4) K_{X'} = 4 K_\alpha$$

$$\therefore \qquad K_{X'} \simeq \frac{4 K_\alpha}{A - 4} \qquad \qquad \dots \dots (25)$$

Putting the value of $K_{X'}$ in equation 24, we get

$$Q \simeq \frac{4 K_\alpha}{A - 4} + K_\alpha$$

$$Q \simeq \frac{A K_\alpha}{A - 4}$$

$$\therefore \qquad K_\alpha \simeq \frac{A - 4}{A} Q \qquad \qquad \dots \dots (26)$$

Putting the value of K_α in equation 25, we get

$$K_{X'} \simeq \frac{4}{A} Q$$

For heavy nuclei, A is large and $A - 4 \simeq A$

Then $K_\alpha \simeq Q$ and $K_{X'} = 0$

This shows that α particles emitted carry off practically all of the disintegration energy available in the form of kinetic energy.

Note: It may be noted that all masses appearing in Q value are nuclear masses but while performing calculation we use standard values of atomic masses. So all nuclear masses must be replaced by atomic masses. In this manipulation electron masses will automatically cancel out.

Thus $Q = [m(X) - m(X') - m(\alpha)] c^2$.

Example 19
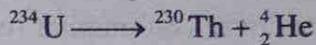
Find the kinetic energy of alpha particle emitted in the decay of ^{234}U.

$m(U) = 234.040952\,u$, $m(Th) = 230.033134\,u$ and $m(\alpha) = 4.002603\,u$.

Solution

Kinetic energy of alpha particle emitted, $K_\alpha = \dfrac{A-4}{A}Q$.

The decay process is

$$^{234}U \longrightarrow \,^{230}Th + \,^4_2He$$

$$\therefore \quad Q = [m(U) - m(Th) - m(\alpha)]c^2$$

$$Q = [234.040952\,u - 230.033134\,u - 4.002603\,u]c^2$$

$$Q = 0.005125\,u\,c^2$$

Using $\qquad 1u = 931.5\dfrac{MeV}{c^2}$

$$Q = 0.005125 \times 931.5\ MeV = 4.858\ MeV$$

$$\therefore \quad K_\alpha = \dfrac{A-4}{A}Q = \dfrac{(234-4)}{234} \times 4.858\ MeV$$
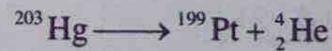
$$K_\alpha = 4.775\ MeV$$

Example 20

Check whether ^{203}Hg undergoes alpha decay. Given $m(Hg) = 202.972872\,u$, $m(Pt) = 198.970593\,u$ and $m(\alpha) = 4.002603\,u$.

Solution

The decay process is

$$^{203}Hg \longrightarrow \,^{199}Pt + \,^4_2He$$

The value is

$$Q = [m(Hg) - m(Pt) - m(\alpha)]c^2$$

$$Q = [202.972872\,u - 198.970593\,u - 4.002603\,u]c^2$$

$$Q = -0.000324 \, u \, c^2$$

$$Q = -0.000324 \times 931.5 \, M \, c^2$$

$$Q = -0.3018 \, MeV.$$

Since Q value is negative, this decay is not permitted.

Quantum theory of alpha decay

In alpha decay of heavy nuclei, two protons and two neutrons come together to form an α particle. As long as α particle is inside the nucleus it is acted upon by short range nuclear force which dominates over coulombian repulsion. Once α particle is ejected out of nucleus it is acted upon by the coulomb force due to daughter nucleus. The variation of potential energy (V) of α particle with distance from the centre of the atom is shown in figure below.
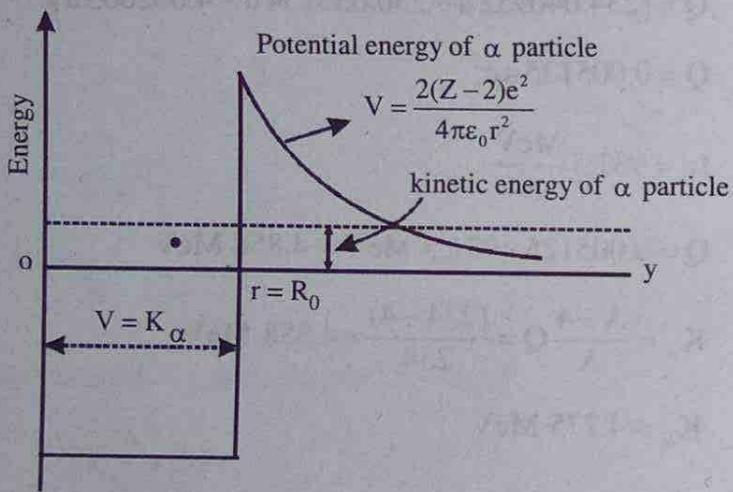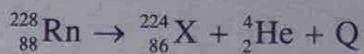
**Figure 1.11**

Outside the nucleus i.e. $r > R_0$, the potential energy V of α particle is due to coloumb force between nucleus of daughter atom and α particle. If Z is the atomic number of parent atom.

$$V = \frac{1}{4\pi\varepsilon_0} 2 \frac{(Z-2)e^2}{r} \qquad \dots\dots (27)$$

where ε_0 is the permittivity of free space. The change from the attractive nuclear force to coulomb force takes place at $r = R_0$, the radius of the nucleus.

Now consider α decay of $^{228}_{88}Rn$. According to the equation

$$\ce{^{228}_{88}Rn} \rightarrow \ce{^{224}_{86}X} + \ce{^{4}_{2}He} + Q$$

The potential energy on the surface of nucleus

$$V_S = \frac{2(Z-2)e^2}{4\pi\varepsilon_0 r} = \frac{9\times10^9 \times 2 \times (88-2) \times (1.6\times10^{-19})^2}{10^{-14}}$$

$$\approx 25 \text{MeV}.$$

But
$$Q = (m_{228} - m_{224} - m_\alpha)c^2$$

$$Q \approx 5 \text{MeV}.$$

This is the energy of the α-particle inside the nucleus. Here Q is much less than V_s. Classically this means that α-particle cannot escape from the nucleus. To escape from the nucleus α-particle must have an energy greater than 25 MeV. But quantum mechanics says that there is a probability of penetration of α-particle through the potential barrier V_s. This is known as the tunnelling effect in quantum mechanics. This idea was firstly put forward by Gamow in 1928 and is known as Gamow's theory of α-decay.

To develop the theory, three assumptions are made. They are

1. An α-particle may exist as an entity within a heavy nucleus.
2. α-particle is in constant motion and is held in the nucleus by a potential barrier.
3. There is a small but definite probability that the particle may tunnel through the barrier (despite its height) each time a collision with it occurs.

According to WKB perturbation theory of barrier penetration, the decay probability per unit time (λ) is given by

$$\lambda = \upsilon T \qquad\qquad\qquad ..... (28)$$

where υ is the number of times per second an alpha particle strikes the potential barrier and T is the transmission probability of the α-particle.

If we assume that at any moment only one alpha particle exists as such in a nucleus and that it moves back and forth along a nuclear diameter.

Thus
$$\upsilon = \frac{1}{t} = \frac{v}{x} = \frac{v}{2R_0}$$

where v is the velocity of the α-particle when eventually leaves the nucleus and R_0 is the radius of the nucleus.

$$\therefore \qquad \lambda = \frac{V}{2R_0} T \qquad\qquad ..... (29)$$

From the theory of tunnel effect, we have

$$T = e^{-2k_2 L} \qquad\qquad ..... (30)$$

with

$$k = \sqrt{\frac{2m(V-E)}{\hbar^2}} \text{ and L is the width of the barrier.}$$

Thus

$$\lambda = \frac{V}{2R_0} e^{-2kL} \qquad\qquad ..... (31)$$

By making suitable rough estimates for the thickness (L) and height of the barrier (V), we can calculate the decay probability λ such that it tallies with the observed λ of isotopes of nuclei.

An exact calculation of the decay probability was first done in 1928 by George Gamow and was one of the first successful applications of the quantum theory.

Beta decay

Beta decay is the process by means of which a nucleus can alter its composition to become more stable. In beta decay, a neutron in the nucleus changes into a proton (or a proton into a neutron); Z and N each change by one unit, but A does not change. The emitted particles, which were called beta particles when first observed in 1898, were soon identified as electrons. There are three types of β-decays observed in nature. These are

1. The negatron emission or β^- decay

2. The positron emission or β^+ decay

3. Orbital electron capture.

We have already discussed these decays in brief. Here we shall discuss some more details of these.

Negatron emission or β^- decay
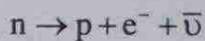
Consider the decay of a neutron into a proton and an electron.

$$n \rightarrow p + e^-$$
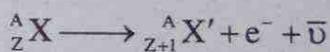
This decay appears to violate the law of conservation of angular momentum that we already discussed. Experiments have shown that the energy emitted by electrons

varies continuously from zero to a maximum value. This implies apparent violation of law of conservation of energy, because all electrons should emerge from the decay $n \rightarrow p + e^-$ with precisely the same energy. Instead, all electrons emerge with less energy but in varying amounts.

To overcome these difficulties in 1930 Wolfgang Pauli suggested that there is a third particle emitted in the decay process. Since the charge is already conserved the third particle emitted must be chargeless. In order to preserve the conservation of angular momentum, the third particle must have spin $\frac{1}{2}$. The missing energy will be carried by the third particle. The new particle is named neutrino (little neutral one in Italian). Every particle has an antiparticle, the antiparticle of neutrino (υ) is antineutrino ($\bar{\upsilon}$). It is in fact an antineutrino is emitted in the decay process. The complete decay process is

$$n \rightarrow p + e^- + \bar{\upsilon}$$

Since the emitted electron has negative charge, this electron is called negatron and the process is called negatron emission or simply β^--decay. Consider a nucleus $_{Z}^{A}X$ undergoes a β^- decay giving a new nucleus $_{Z+1}^{A}X'$:

$$_{Z}^{A}X \longrightarrow _{Z+1}^{A}X' + e^- + \bar{\upsilon}$$

Using law of conservation of energy

$$m_X c^2 = m_{X'} c^2 + m_e c^2 + K_{X'} + K_e + K_{\bar{\upsilon}}$$

Here m_X and $m_{X'}$ are nuclear masses and $K_{X'} + K_e$ are kinetic energies of new nucleus X' and emitted electron respectively. $K_{\bar{\upsilon}}$ is the kinetic energy of the antineutrino. Usually the recoil kinetic energy $(K_{X'})$ of the nucleus X' is negligibly small. So the Q value is:

$$Q = K_e + K_{\bar{\upsilon}}$$

Thus the above equation becomes

$$m_X c^2 = m_{X'} c^2 + m_e c^2 + Q$$

or

$$Q = (m_X - m_{X'} - m_e)c^2 \qquad\qquad ..... (32)$$

This the expression for the Q-value of β^--decay process.

To do the calculations we convert nuclear masses into atomic masses.

$$m_X = m(X) - Zm_e \quad \text{and} \quad m_{X'} = m(X') - (Z+1)m_e$$

where $m(X)$ and $m(X')$ are atomic masses.

Putting the values of m_X and $m_{X'}$ in equation 32, we get

$$Q = \left[m(X) - Zm_e - m(X') + (Z+1)m_e - m_e\right]c^2$$

or $\qquad\qquad Q = \left[m(X) - m(X')\right]c^2 \qquad\qquad\qquad ..... (33)$

This is the expression for the Q-value of β^--decay process in terms of atomic masses.

Example 20

Consider β^--decay represented by $^{15}_{6}C \longrightarrow ^{15}_{7}N + e^- + \bar{\upsilon}$

Given $m(C) = 15.010599\,u$, $m(N) = 15.000109\,u$. Calculate the Q-value.

Solution

The Q-value of the process is

$$Q = [m(C) - m(N)]c^2$$

$$Q = [15.010599\,u - 15.000109\,u]c^2$$

$$Q = 0.01049 \times u\,c^2$$
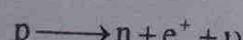
$$Q = 0.01049 \times 931.5\,\text{MeV}$$

$$Q = 9.771435\,\text{MeV}$$

Since Q is positive this process is possible.
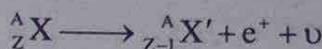
Positron emission or β^+ decay

In this process a proton decays into a neutron, positive electron (positron) and a neutrino. The process is given by

$$p \longrightarrow n + e^+ + \upsilon$$

Since it emits a positron, the process is called positron emission or β^+-decay. In this process the proton number decreases by one but A remains unchanged.

Consider a nucleus $_Z^AX$ undergoes a β^+-decay giving a new nucleus $_{Z-1}^AX'$:

$$_Z^AX \longrightarrow _{Z-1}^AX' + e^+ + \upsilon$$

Using law of conservation of energy

$$m_X c^2 = m_{X'} c^2 + m_e c^2 + K_{X'} + K_{e^+} + K_\upsilon$$

where m_X and $m_{X'}$ are nuclear masses. $K_{X'}$, K_{e^+} and K_υ are the kinetic energies of the nucleus after decay, positron and neutrino respectively. Usually the recoil kinetic energy $(K_{X'})$ of the nucleus (X') is negligibly small. So the Q-value is:

$$Q = K_{e^+} + K_\upsilon$$

Thus the above equation becomes

$$m_X c^2 = m_{X'} c^2 + m_e c^2 + Q$$

$\therefore$
$$Q = (m_X - m_{X'} - m_e)c^2 \qquad \qquad \text{..... (34)}$$

This is the expression for the Q-value of β^+-decay process.

Now we convert nuclear masses into atomic masses.

$$m_X = m(X) - Zm_e$$

and
$$m_{X'} = m(X') - (Z-1)m_e$$
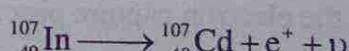
where $m(X)$ and $m(X')$ are the atomic masses. Putting the values of m_X and $m_{X'}$ in equation (34), we get

$$Q = [m(X) - Zm_e - m(X') + (Z-1)m_e - m_e]c^2$$

$$Q = [m(X) - m(X') - 2m_e]c^2 \qquad \qquad \text{..... (35)}$$

This is the expression for Q-value of β^+-decay process in terms of atomic masses.

Example 21

Consider the β^+-decay represented by

$$_{49}^{107}In \longrightarrow _{48}^{107}Cd + e^+ + \upsilon$$

Given that $m(In) = 106.918263\,u$

$m(Cd) = 106.906618 \, u \quad m_e = 0.0005486 \, u$. Calculate the Q-value

Solution

The Q-value of the process is

$$Q = [m(In) - m(Cd) - 2m_e]c^2$$
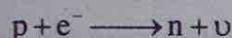
$$Q = [106.918263 \, u - 106.906618 \, u - 2 \times 0.0005486 \, u]c^2$$

$$Q = 9.825276 \text{ MeV}$$
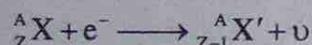
Since Q is positive, this process is possible.

Electron capture

In this process the nucleus absorbs one of the orbital electrons of the atom. The electrons nearest to the nucleus are in K-shell. Therefore the nucleus is most likely to absorb an electron in K-shell. For this reason this capture is known as K-capture. The probability of a L or M shell electron to be absorbed by nucleus is much less than the probability of K-shell capture. The basic electron capture process is

$$p + e^- \longrightarrow n + \upsilon$$

In a nucleus K-capture can be represented as

$$^A_Z X + e^- \longrightarrow ^A_{Z-1} X' + \upsilon$$

In this process the proton number decreases by one and A remains the same. Using law of conservation of energy, we have

$$m_X c^2 + m_e c^2 + K_e = m_{X'} c^2 + K_{X'} + K_\upsilon$$

Neglecting initial kinetic energy of the electron and the recoil kinetic energy of the nucleus, the Q-values energy of the nucleus, the Q-values becomes:

$$Q = K_\upsilon$$

Thus we have

$$m_X c^2 + m_e c^2 = m_{X'} c^2 + Q$$

$$\therefore \qquad Q = [m_X + m_e - m_{X'}]c^2 \qquad \qquad ..... (36)$$

This is the expression for Q-value of the electron capture process.

Now convert nuclear masses into atomic masses

$$m_X = m(X) - Zm_e$$

$$m_{X'} = m(X') - (Z-1)m_e$$

Putting the value of m_X and $m_{X'}$ in equation 36, we get

$$Q = [m(X) - Zm_e + m_e - m(X') + (Z-1)m_e]c^2$$

$$Q = [m(X) - m(X')]c^2 \qquad\qquad\qquad ..... (37)$$

This is the expression for the Q-value of electron capture process in terms of atomic masses.

In contrast to beta-decay processes, a mono energetic neutrino is emitted in electron capture. It may also be noted that if $m(X) > m(X')$ electron capture will occur.

If $m(X) > m(X') + 2m_e$, β^+-decay process occurs.

Example 22

^{75}Se decays by electron capture to ^{75}As. Find the energy of the emitted neutrino. $m(\text{Se}) = 74.922523\,u$, $m(\text{As}) = 74.921596\,u$.

Solution

The Q-value of electron capture = energy of emitted neutrino

$$Q = [m(\text{Se}) - m(\text{As})]c^2$$

$$Q = 74.922523\,u - 74.921596\,u]c^2$$

$$Q = 0.8635\,\text{MeV}.$$

Gamma decay

Like atoms, nuclei also have excited states. When a nucleus jumps from an excited state to ground state, it emits a photon (γ-ray) whose energy is equal to the energy difference between the excited state and the ground state. The energies of emitted gamma rays are typically in the range of 100 KeV to a few MeV.

When a nucleus emits an alpha particle or beta particle, the final nucleus may be left in an excited state. Most excited nuclei have very short half lives in the order of 10^{-9}s to 10^{-12}s. After this the excited nucleus comes back to the ground state after emitting one or two photon known as nuclear gamma rays.

There are few cases of excited states with half lives of hour, days or even years.

Along lived excited state is called an isomer of the same nucleus in its ground state. The excited nucleus $\left({}^{87}_{38}Sr\right)^*$ has a half life of 2.8 hours. Thus Sr^* is an isomer of Sr.

Similarly $\left({}^{79}_{35}Br\right)^*$ and $\left({}^{81}_{25}Br\right)^*$ are isomeric states of ${}^{80}_{35}Br$.

Consider a nucleus jumps from an initial state with energy E_i to final state of energy E_f emitting a γ-ray of energy E_γ. According to law of conservation of energy, we have

$$E_i = E_f + E_\gamma + K_R$$

where K_R is the recoil kinetic energy of the nucleus. Usually K_R is negligibly small.

Thus
$$E_i = E_f + E_\gamma$$

or
$$E_\gamma = E_i - E_f$$

This shows that the gamma-ray energy is equal to the difference between the initial and final energy states.

Assume that before the emission, nucleus is at rest so its momentum zero. When its a γ-ray of momentum p_γ normally nucleus will recoil to conserve momentum.

Let p_R be the recoil momentum of the nucleus. Conservation of momentum gives

$$0 = \vec{p}_R + \vec{p}_\gamma$$

or
$$\vec{p}_R = -\vec{p}_\gamma$$

The recoil energy $K_R = \dfrac{p_R^2}{2m}$

where m is the mass of the nucleus

or
$$K_R = \frac{p_\gamma^2}{2m} \qquad (\because \quad p_R = -p_\gamma)$$

or
$$K_R = \frac{p_\gamma^2 c^2}{2mc^2} = \frac{(p_\gamma c)^2}{2mc^2} = \frac{E_\gamma^2}{2mc^2}$$

If $E_\gamma = 1\,\text{MeV}$ and $m = 100\,\text{u}$

$$K_R = \frac{1(\text{MeV})^2}{2 \times 100\,\text{u}\,c^2}$$

$$1\,\text{u} = 931.5\,\frac{\text{MeV}}{c^2}$$

$$\therefore \quad K_R = \frac{(\text{MeV})^2}{2 \times 100 \times 931.5\,\text{MeV}}$$

$$K_R = \frac{\text{MeV}}{200 \times 931.5} = \frac{10^6\,\text{eV}}{200 \times 931.5} = 5.368\,\text{eV}$$

This is obviously negligibly small.

Example 23

The nucleus ^{198}Hg has excited states at 0.412 and 1.088 MeV. Following the beta decay of ^{198}Au to ^{198}Hg, three gamma-rays are emitted. Find the energies of these three gamma rays.

Solution

E_3	_____	1.088 MeV

The energies of the emitted gamma rays are

E_2	_____	0.412 MeV
E_1	_____	0 MeV

$$\Delta E_{21} = E_2 - E_1 = 0.412 - 0 = 0.412\,\text{MeV}$$

$$\Delta E_{31} = E_3 - E_1 = 1.088 - 0 = 1.088\,\text{MeV}$$

$$\Delta E_{32} = E_3 - E_2 = 1.088 - 0.412 = 0.676\,\text{MeV}$$

Here we neglected the small recoil energy.

Example 24

Compare the recoil energy of a nucleus of mass 200 that emits (a) a 5.0 MeV alpha particle and (b) a 5.0 MeV gamma ray.

Solution

(a) When a nucleus emits an alpha particle its Q-value is:

$$Q = K_{X'} + K_\alpha$$

where $K_{X'}$ is the recoil energy of the nucleus

$$\therefore \qquad K_{X'} = Q - K_\alpha$$

Using
$$Q = \frac{A}{A-4} K_\alpha$$

$$K_{X'} = \frac{A}{A-4} K_\alpha - K_\alpha = \frac{4}{A-4} K_\alpha$$

$$K_{X'} = \frac{4 \times 5}{200-4} = \frac{20}{196} = 0.102 \text{ MeV}.$$

(b) The gamma-ray recoil energy is given by:

$$K_R = \frac{E_\gamma^2}{2mc^2} = \frac{(5.0 \text{ MeV})^2}{2 \times 200 \, u \, c^2}$$

$$1 u = 931.5 \text{ MeV}/c^2$$

$$K_R = \frac{(5.0 \text{ MeV})^2}{400 \times 931.5 \dfrac{\text{MeV}}{c^2} \cdot c^2}$$

$$K_R = \frac{25}{400 \times 931.5} \text{ MeV}$$

$$K_R = 67.096 \text{ eV}.$$

Natural radioactivity

The radioactivity exhibited by naturally occurring elements is called natural radioactivity. It takes place spontaneously without any external causation i.e. no cause effect relationship (causality) is involved in the decay process. It is exhibited by the nuclei with $Z > 82$. Some examples are radium, actinium, polonium, etc. The ultimate product of the natural radioactivity is lead with $Z = 82$.

Certain nuclei which are stable can be converted into unstable isotopes by bombarding them with fast moving particles. The unstable isotope so formed undergo radioactive decay. Thus, the phenomenon of radioactive decay by artificial means is called artificial radioactivity. The half life of the artificially produced isotope may

be short and the decay may stop as soon as the bombardment with fast moving particles ceases. The phenomenon of artificial radioactivity that persists long after the bombardment with fast moving particles ceases is called induced radioactivity.

Cause of radioactivity

Unstability of nucleus is the main cause of radioactivity. We know that the nuclei with higher atomic number have low binding energy per nucleon. Also due to the presence of large number of protons in the nucleus, the Coulombian repulsion is also stronger. Owing to these three reasons nuclei with $Z > 82$ spontaneously decay emitting radiations so as to achieve stability.

Cause of natural radioactivity surrounds us

All of the elements beyond the very lightest (hydrogen and helium) were produced by nuclear reactions in the interiors of stars. These reactions not only produce stable elements but also radioactive ones. Most radioactive elements have half-lives that are much smaller than the age of the earth (about $4.5 \times 10^9 y$), so those radioactive elements that may have been present when the earth was formed have decayed to stable elements. However, a few of the radioactive elements created long ago have half-lives that are greater than the age of the earth. These elements can still be observed to undergo radioactive decay and account for part of the back ground of natural radioactivity that surrounds.

Radioactive series

When a radioactive substance (parent) emits an α or β particle an entirely new substance (product) is left which is still radioactive and sooner or later emit another particle to become a still different atom. The product has different chemical and physical properties from the parent. This process continues through a series of elements ending up finally with an atom which is stable and not radioactive. This is called radioactive series.

It has been experimentally observed that most of the radioactive substances found in nature are members of four series namely thorium series, neptunium series, uranium series and actinium series. The mass numbers of above four known series could be represented by the following set of numbers. $4n$ for the thorium series, $4n + 1$ for the neptunium series, $4n + 2$ for the uranium series and $4n + 3$ for the actinium series, where n is an integer, see the table below. It may also be noted that since the half life of neptunium is so small compared with the age of the solar system that members of the series are not found on the earth today.

Four Radioactive Series

Mass Numbers	Series	Parent	Half-Life Years	Stable end Product
4n	Thorium	$^{232}_{90}\text{Th}$	1.39×10^{10}	$^{208}_{82}\text{Pb}$
4n + 1	Neptunium	$^{237}_{93}\text{Np}$	2.25×10^{6}	$^{209}_{83}\text{Bi}$
4n + 2	Uranium	$^{238}_{92}\text{U}$	4.47×10^{9}	$^{206}_{82}\text{Pb}$
4n + 3	Actinium	$^{235}_{92}\text{U}$	7.07×10^{8}	$^{207}_{82}\text{Pb}$

It may be noted that the neptunium series (4n + 1) begins with ^{237}NP, which has a half life of only 2.25×10^6 y which is less than the age of the earth 4.5×10^9 y. Thus all of the ^{237}Np might have decayed to $^{209}_{83}\text{Bi}$.

One important use of study of natural radioactivity is to find the ages of the rocks there by that of earth. If we examine a sample of uranium bearing rock, we can find the ratio of ^{238}U atoms to ^{206}Pb atoms. From this we can estimate the age of the rock. See example given below.

Example 25

In a rock the ratio of uranium to lead is found to be one. Estimate the age of the rock. $t_{1/2} = 4.5 \times 10^9$ y

Solution

Let N_0 be the original number of uranium atoms present. $N(N_0 e^{-\lambda t})$ be the number of uranium atoms present now. It means that $N_0 - N$ atoms have been decayed into Pb and presently observed as ^{206}Pb. Thus the ratio (R) of uranium to lead is

$$R = \frac{\text{Number of } ^{238}\text{U present now}}{\text{Number of } ^{206}\text{Pb present now}}$$

i.e.

$$R = \frac{N_0 e^{-\lambda t}}{N_0 - N_0 e^{-\lambda t}}$$

or

$$R = \frac{1}{e^{\lambda t} - 1}$$

$$e^{\lambda t} - 1 = \frac{1}{R}$$

$$e^{\lambda t} = \frac{1}{R} + 1$$

Taking log on both sides, we get

$$\lambda t = \ln\left(1 + \frac{1}{R}\right)$$

$$t = \frac{1}{\lambda} \ln\left(1 + \frac{1}{R}\right)$$

Using
$$\lambda = \frac{0.693}{T_{1/2}}$$

$$t = \frac{T_{1/2}}{0.693} \ln\left(\frac{1}{R} + 1\right)$$

$$\therefore \quad t = \frac{4.5 \times 10^9}{0.693} \ln(1+1) = \frac{4.5 \times 10^9 \times \ln 2}{0.693}$$

$$t = 4.5 \times 10^9 \text{ years.}$$

Thus the age of the rock is 4.5×10^9 years.

Example 26

The radioactive decay of ^{232}Th leads eventually to ^{208}Pb. A certain rock is examined and found to contain 3.65 grams of ^{232}Th and 0.75 grams of ^{208}Pb. Assuming all of the Pb was produced in the decay of Th, what is the age of the rock. $T_{1/2}$ of thorium is 1.41×10^{10} y.

Solution

Number of atoms contained in 3.65 g of thorium,

$$N_{Th} = \frac{3.65}{232} \times N_A$$

Number of atoms contained in 0.75 g of ^{208}Pb,

$$N_{Pb} = \frac{0.75}{208} \times N_A$$

$$\therefore \quad R = \frac{N_{Th}}{N_{Pb}} = \frac{3.65 \times 208}{232 \times 0.75} = \frac{759.2}{174}$$

$$R = 4.363.$$

Using
$$t = \frac{T_{1/2}}{0.693} \ln\left(\frac{1}{R}+1\right)$$

$$t = \frac{1.41 \times 10^{10}}{0.693} \ln\left(\frac{1}{4.363}+1\right)$$

$$t = 4.198 \times 10^9 \text{ years.}$$

Mossbauer effect

Mossbauer effect is the phenomenon which involves the resonant and recoil free emission and absorption of gamma-rays by atomic nuclei bound in a solid.

This phenomenon was discovered by German physicist Rudolf Mossbauer in the year 1958. For this Mossbauer was awarded the 1961 Nobel Prize in physics.

One way of studying atomic systems is to do resonance experiments. Resonance is the phenomenon of overlapping of the emission and absorption spectrum. In resonant experiments what we do is radiation from a collection of atoms in an excited state is allowed to incident on a collection of identical atoms in their ground state. The ground state atoms can absorb the emitted photons by the former one and jump to the corresponding excited state. In principle resonant absorption will not occur. This is because when atom emits a photon, it is due to the recoil of the atom, the photons energy is less than the transition energy by the recoil kinetic energy. The absorbing atom also recoil. Hence photons energy is less than the transition by $2K_R$. So normally expect that absorption will not occur. However, the absorption is still possible. This is because the excited states don't have exact energies. According to uncertainty principle $\Delta E \Delta t \approx \hbar$, the state lives for time Δt and during that time we can't determine its energy to an accuracy less than ΔE. That is the energy spectrum spreads over a width ΔE.

Since K_R is much less than the width ΔE the emission spectrum and the absorption spectrum can overlap there by occurring absorption process. See figure (a) given below.

In the case of nuclear gamma rays the situation is different. A typical life is 10^{-10} s, so width

$$\Delta E = \frac{\hbar}{10^{-10}} \simeq 10^{-5} \text{ eV.}$$

i.e. the width of energy spread is very narrow. For γ-rays

$$E_\gamma = E_i - E_f + K_R$$

If $E_\gamma = 10^5 \, eV$ we found that $K_R = 1eV$ so $2K_R = 2eV$

This shows that, since $2K_R (2eV)$ is much larger than $\Delta E(10^{-5} eV)$ no overlapping is possible. So resonance absorption cannot occur. See figure (b) below.
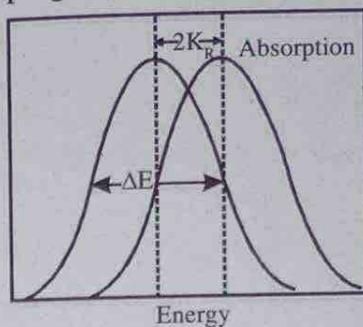
**Figure 1.12(a): Emission and absorption energies in an atomic system**
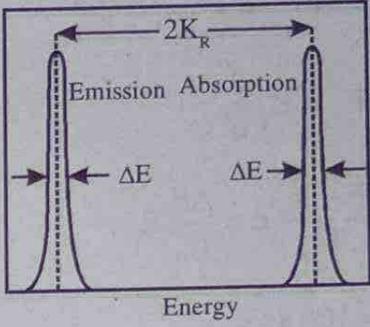
**Figure 1.12(b): Emission and absorption energies in a nuclear system**
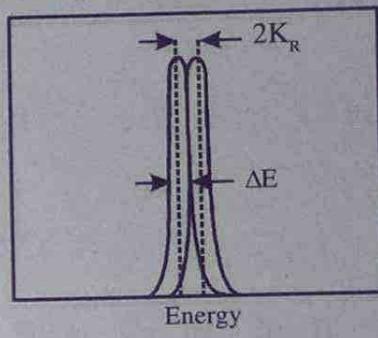
Figure 1.12(c): Absorption energies for nuclei bound in crystal lattice

For the overlapping to occur we have to make recoil kinetic energy as small as possible. The recoil kinetic energy of gamma ray is given by $K_\gamma = \dfrac{E_\gamma^2}{2mc^2}$. To make K_γ smaller m has to be made larger. This is what Mossbauer did. Mossbauer placed the radioactive nuclei and the absorbing nuclei in crystals. The crystalline binding energies are large compared with K_R, so the individual atoms are held tightly to their position in the crystal lattice and are not free to recoil. When gamma-ray radiation occurs in the crystal, the whole crystal that recoils. This makes m in the expression for K_γ larger, may be 10^{20} times larger than the atomic mass. When the recoil energy is very small resonant absorption occurs. That is emission spectrum and absorption spectrum overlaps. See figure (c) above.

To obtain a complete overlap we have to shift either the emission or absorption energies. This can be done by Doppler shift. According to Doppler effect

$$\upsilon = \upsilon_0 \left(1 + \frac{v}{c}\right)$$

when a source of frequency υ_0 moving towards the observer with a speed v. The above equation can be written as

$$h\upsilon = h\upsilon_0\left(1 + \frac{v}{c}\right)$$

$$E = E_0\left(1 + \frac{v}{c}\right)$$

or

$$E - E_0 = E_0\frac{v}{c}$$

$$v = \frac{E - E_0}{E_0}c = \frac{\Delta E}{E_0}E$$

If $\Delta E = 10^{-5}\,eV$ and $E_0 = 100\,keV$

we get

$$v = \frac{10^{-5}}{10^5}\times 3\times 10^8 = 3\,cm/s.$$

This shows that when a gamma-ray emitting source is moving towards a gamma ray absorbing source with a speed 3 cm/s, the spreads will overlap.

The Mossbauer effect can be used as a probe to observe interactions between a nucleus and its electrons. This is because gamma-rays have very narrow line width. This means that this is very sensitive to small changes in energies of nuclear transitions. Thus the Mossbauer effect is an extremely precise method for measuring small changes in the energies of photons.

IMPORTANT FORMULAE

1. Nuclear radius, $R = R_0 A^{1/3}$, where $R_0 = 1.2\,fm$

2. Nuclear binding energy:

$$E_b = [Nm_n + Zm(^1_1H) - m(X)]c^2$$

where $m(^1_1H)$ and $m(X)$ are atomic masses.

3. Proton separation energy:

$$S_p = \left[m\left(^{A-1}_{Z-1}X'\right) + m\left(^1_1H\right) - m\left(^A_ZX\right)\right]c^2$$

4. Neutron separation energy:

$$S_n = \left[m\left(^{A-1}_{Z-1}X\right) + m_n - m\left(^A_ZX\right)\right]c^2$$

5. Semiempirical binding energy formula:

$$E_b = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a \frac{(A-2Z)^2}{A} + E_p$$

6. Most stable isobar:

$$Z = \frac{4a_a A + a_c A^{2/3}}{2a_c A^{2/3} + 8a_a}$$

7. Radioactive decay law:

$$N = N_0 e^{-\lambda t}, \quad \text{where} \quad \lambda = \frac{0.693}{T_{1/2}}$$

8. The number of nuclei after n half lives:

$$N = \frac{N_0}{2^n}$$

9. Activity of a radioactive sample:

$$R = \frac{dN}{dt} = \lambda N$$

or

$$R = \frac{0.693}{T_{1/2}} \frac{mN_a}{A}$$

10. Expression for mean life time

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$$

11. Age of the sample (wood):

$$t = \frac{1}{\lambda} \ln\left(\frac{R_0}{R}\right)$$

R_0 is the initial activity and R is the present activity.

12. Q-value of alpha decay:

$$Q = [m(X) - m(X') - m(\alpha)]c^2$$

where

$$Q = K_{X'} + K_\alpha$$

$$K_\alpha \simeq \frac{A-4}{A} Q.$$

13. Q-values of beta decays:

a) β^--decay

$$Q = [m(X) - m(X')]c^2$$

where $$Q = K_{X'} + K_e + K_{\bar{\upsilon}}$$

b) β^+-decay

$$Q = [m(X) - m(X') - 2m_e]c^2$$

where $$Q = K_{X'} + K_{e^+} + K_{\upsilon}$$

14. Q-value of electron capture:

$$Q = [m(X) - m(X')]c^2$$

where $Q \simeq K_{\upsilon}$ neglecting K_e and $K_{X'}$

15. Recoil energy of gamma-decay:

$$K_R = \frac{E_\gamma^2}{2mc^2}$$

16. Expression for the age of the rock:

$$t = \frac{T_{1/2}}{0.693} \ln\left(\frac{1}{R} + 1\right)$$

where R is the ratio number of decaying atoms at present and the number of decayed atom.

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in* **two** *or* **three** *sentences*)

Short answer type questions

1. What is a nucleus? Give three of its properties.
2. What is the origin of nuclear force?
3. What are the similarities between atomic and nuclear structure.
4. Write down two major differences between the study of nuclei and atoms.
5. Why the study of nuclear structure is important?
6. What are isotopes? Give an example.
7. What are isotones? Give an example.
8. What are isobars? Give an example.

9.   What are nuclear electrons?

10. What is proton-electron model of nucleus?

11. What is proton-neutron hypothesis?

12. Draw the graphical variation of nuclear density with radius of nuclei.

13. Define binding energy of a nucleus.

14. What is a binding energy curve?

15. What is meant by proton separation energy?

16. What is meant by neutron separation energy?

17. Give the name of three nuclear models.

18. Distinguish between strong interaction model and independent particle model.

19. Write down the semiempirical mass formula and explain the symbols used.

20. What is volume energy of a nucleus?

21. What is surface energy of a nucleus?

22. What is Coulomb energy of a nucleus?

23. What is shell model?

24. What are magic numbers?

25. What is the cause of magic numbers?

26. What is nuclear force?

27. Define mass defect and binding energy.

28. Why is the binding energy curve steep for lighter nuclei and falling off slowly for the heavy nuclei?

29. Account for the unstability of heavy nuclei.

30. Why the nuclei, that are off the line of stability, unstable?

31. What is the density of the nucleus to that of water?

32. Why is N approximately equal to Z for stable nuclei?

33. Why N is greater than Z for heavy nuclei?

34. Why there are no stable isotopes with Z > 83?

35. What is meant by radioactivity?

36. State the law of radioactive disintegration.

37. Distinguish between natural and artificial radioactivity.

38. What is the cause of radioactivity?

39. Define radioactive decay constant.

40. What is alpha decay? Give an example.

41. What is beta decay? Give an example.

42. What is gamma decay? Give an example.
43. What is meant by electron capture?
44. What is meant by positron emission?
45. What is meant by activity of a radioactive sample? What is its unit?
46. Define Q-value of a decay process.
47. Write down expression for Q-value of the alpha decay process.
48. Write down an expression for Q-value of the β^--decay process.
49. Write down an expression for Q-value of the β^+ decay.
50. Write down an expression for Q-value of the electron capture process.
51. What is meant by inverse β-decay?
52. What is gamma-decay?
53. What is an isomer?
54. What is meant by natural radioactivity?
55. Distinguish between natural and artificial radioactivity.
56. What is meant by induced radioactivity?
57. What is the cause of radioactivity?
58. What is radioactive series? Give their names.
59. What is Mossbauer effect?
60. What is the use of Mossbauer effect?

Section B
*(Answer questions in a **paragraph** of about half a page to one page)*

Paragraph / Problem type questions

1. Give three evidences against the idea of nuclear electrons.
2. How did proton-electron hypothesis fail?
3. Justify that electrons cannot exist inside a nucleus using the uncertainty principle.
4. Give the symbol for the following.
   a) The isotope of helium with mass number 4.
   b) The isotope of tin with 66 neutrons.
   c) An isotope with mass number 235 that contains 143 neutrons.

   $\left[\text{a) } {}_{2}^{4}\text{He}, \quad \text{b) } {}_{50}^{116}\text{Sn} \quad \text{b) } {}_{92}^{235}\text{Sn} \right]$

5. How can you estimate the radius of a nucleus?
6. Arrive at an expression for binding energy of a nucleus.

7. What are the informations that we can obtain from a binding energy curve?

8. Arrive at an expression for proton separation energy.

9. Arrive at an expression for neutron separation energy.

10. Explain the liquid drop model.

11. What are the factors on which the binding energy of a nucleus depend according to liquid drop model?

12. Explain the assymmetry energy of a nucleus.

13. Explain the pairing energy of a nucleus.

14. What are the assumptions on which shell model is based?

15. What are the merits and demerits of liquid drop model?

16. What are the merits and demerits of shell model?

17. Give four properties of nuclear force.

18. Briefly explain nuclear force model proposed by Yukawa.

19. Roughly estimate the mass of the pion using uncertainty principle.

20. Why do we say that a nucleus behaves like a drop of a liquid?

21. What is meant by half life? Derive an expression for it.

22. What is meant by meanlife. Derive an expression for it.

23. Explain radioactive carbon dating.

24. Explain the tunnel theory of alpha decay in brief.

25. What are the assumptions made in the theory of alpha decay.

26. Explain how does β-decay apparently violate law of conservation of energy.

27. Explain the apparent violation of conservation of linear momentum in β-decay.

28. Explain the apparent violation of law of conservation of angular momentum in β-decay.

29. In β-decay, law of conservation of energy, linear momentum, angular momentum are apparently violated. How it is overcome?

30. What is neutrino hypothesis?

31. Derive expression for the recoil energy of a nucleus which emitted a gamma-ray.

32. How will you estimate the age of a rock by using radioactivity?

33. What is meant by radioactive carbon dating?

34. Give a brief account of Mossbauer effect.

35. How to achieve complete overlap of emission spectrum and absorption spectrum?

36. Compute the density of a typical nucleus and find the resultant mass if we could produce a nucleus with a radius of 1 cm.

$$[2 \times 10^{17} \text{kgm}^{-3}, \ 8 \times 10^{11} \text{kg}]$$

37. Compute the Coulomb repulsion energy between two nuclei of ^{16}O that just touch at their surfaces.   [15.26 MeV]

38. Find the total binding energy and the binding energy per nucleon for $^{133}_{55}Cs$. Given that

    $m_n = 1.008665\,u$, $m(^1_1H) = 1.007825\,u$ and $m(Cs) = 132.905452\,u$

    [1118.53868 MeV, 8.410065 MeV per nucleon]

39. Find the proton separation energy of ^{40}Ca. Given that $m(^1_1H) = 1.007825\,u$,

    $m(^{39}K) = 38.963707\,u$, $m(^{39}Ca) = 39.962591\,u$ [8.3285415 MeV]

40. Find the neutron separation energy of ^7Li. Given: $m_n = 1.008665\,u$,

    $m(^6Li) = 6.015123\,u$ $m(^7Li) = 7.016005\,u$.   [7.249865 MeV]

41. Which isobar of $A = 75$ does the liquid drop model suggest is most stable $a_a = 19$ and

    $a_c = 0.595$. $\left[^{75}_{33}As \right]$

42. Tritium ^3_1H has halflife of 12.5 years against β-decay. What fraction of the sample of pure tritium will remain unchanged after 25 years $\left[\dfrac{1}{4} \right]$

43. The half life of $^{238}_{92}U$ against alpha decay is 4.5×10^9 years. Calculate the time taken by uranium to reduce 80% of the initial amount   [1.45×10^9 years]

44. A radioactive isotope X has a half life of 3 second. Initially a sample of this isotope contains 8000 atoms. calculate
    a) Its decay constant
    b) The time t_1 when 1000 atoms of the isotope X remain in the sample.
    c) The number of decay per second in the sample at $t = t_1$

    $\left[\begin{array}{l} \text{a) } 0.231 \text{ s}^{-1} \\ \text{b) } 9\text{s} \\ \text{c) } 231 \text{ s}^{-1} \end{array}\right]$

45. The mean lives of a radioactive substances are 1620 year and 405 year for alpha emission and beta emission respectively. Find the time during which three fourth of a sample will decay if it is decaying both by alpha and beta emission simultaneously

    [449 years]

46. Calculate the mass in gram of a radioactive sample ^{214}Pb having an activity of one microcuri and a half life of 26.8 minute.

    [3.04×10^{-14} g]

47. An animal bone fragment found in an archeological site has a carbon mass of 200g. It registers an activity of 16 decays. What is the age of the bone $\lambda = 3.83 \times 10^{-12}$ s^{-1}

[9400 year]

48. A bone suspected to have originated during the period of Asoka the great was found in Bihar. Accelarator technique gives $\dfrac{^{14}C}{^{12}C} = 1.1 \times 10^{-12}$. Is the bone old enough to have belonged to that period.

[No, 719 year]

49. The atomic ratio between the uranium isotopes ^{238}U and ^{234}U in a mineral sample is found to be 1.8×10^4. The half life of ^{234}U is $T_{1/2}(234) = 2.5 \times 10^5$ y. Find the half life of ^{238}U.

[4.5×10^9 y]

50. A laboratory has 1.49μg of $^{13}_{7}$N, which has a half life of 10 minutes. (a) How many nuclei are present initially (b) What is the activity initially (c) What is the activity after 1 hour. After how long will the activity drop to less that one per second

[a) 6.9×10^{16}, b) 8×10^{13} s^{-1}, c) 1.23×10^{12} s^{-1}, d) 2.76×10^4 s]

51. A rock sample contains 1mg of ^{206}Pb and 4mg of ^{238}U whose half life is 4.47×10^9 y. How long ago the rock formed.

[1.64×10^9 y]

52. The activity R of a sample of a unknown radio nuclide is measured at hourly intervals. The results in MBq are 80.5 and 36.2. Find the half life of the radio nuclide

[52 minutes]

53. Calculate the approximate mass of uranium which must undergo fission to produce the same energy as 100 tonnes of coal. Heat combustian of coal 8000 cal/g. 1 calorie = 4J. Fission of uranium releases 200 MeV

[39.2g]

54. Find the kinetic energy of the alpha particle emitted in the decay process

$$^{226}Ra \longrightarrow ^{222}Ra + {}^{4}_{2}He$$

$m(Ra) = 226.025410$ u, $m(Rn) = 222.0175778$ u and $m(\alpha) = 4.002603$ u

[4.7846 MeV]

55. Check whether the decay processes allowed or not

$$^{211}At \longrightarrow ^{207}Bi + \alpha$$

$m(At) = 210.987496$ u, $m(Bi) = 206.978471$ u $m(\alpha) = 4.002603$ u   [allowed]

56. Find the maximum kinetic energy of the electrons emitted in the negative β-decay of ^{11}Be. Given: $m(Be) = 11.021658$ u $m(\beta) = 11.009305$ u

[11.506 MeV]

57. ^{23}Ne decays to ^{23}Na by negative β-emission what is the maximum kinetic energy of the emitted electrons. Given: m(Ne) = 22.994467 u, m(Na) = 22989769 u

[4.376 MeV]

58. ^{40}K isotope decays by electron capture. Find the Q-value. Given: m(K) = 39.964998u m(Ar) = 39.962383 u.

[1.504 MeV]

59. ^{12}N beta decays to an excited state of ^{12}C, which subsequently decays to the ground state with the emission of a 4.43 MeV gamma-ray. What is the maximum kinetic energy of the emitted beta particle. Given: m(C) = 12.000000 u, m(N) = 12.018613 u $m_e = 0.000549$ u.

[11.89 MeV]

60. The 4n radioactive decay series begins with $^{232}_{90}$Th and ends with $^{248}_{82}$Pb.

 a) How many alpha decays are in the series
 b) How many beta decays
 c) How much energy is released in the complete series. m(Th) = 232.038050u,

 $m(Pb) = 207.976636$ u and $m_\alpha = 4.002603$ u   [a) 6 b) 4 c) 42.658974 MeV]

Section C
(Answer questions in about **two** *pages)*

Long answer type questions (Essays)

1. Explain the postulates of a liquid drop model. Derive Weizsacker semi empirical mass formula.
2. Disscuss the various factors which contribute to the binding energy of nuclei and derive a formula for atomic mass M of a nucleus based on these considerations.
3. Give salient features of nuclear shell model and point out its success and failures.

Hints to problems

36. $\rho = \dfrac{m}{V} = \dfrac{Am_p}{\dfrac{4}{3}\pi R^3}$, $R = 1.2 \times A^{1/3}$ fm

 $m_p = 1.67 \times 10^{-27}$ kg, get ρ.

 $m = \rho V = 2 \times 10^{17} \times \dfrac{4}{3}\pi(0.01)^3 = 8 \times 10^{11}$ kg

37. Radius of oxygen nucleus

 $$R = R_0 A^{1/3} = 1.2 \times (16)^{1/3} = 3.02\,\text{fm}$$

 Coulomb energy, $U = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r}$

 $q_1 = 8e,\ q_2 = 8e,\ r = 2R = 6.04$ fm

$$U = \frac{9 \times 10^9 \times 64 \times e^2}{(6.04 \times 10^{-15})}$$

$$U = \frac{9 \times 10^9 \times 64 \times (1.6 \times 10^{-19})^2}{(6.04 \times 10^{-15})} J$$

or $$U = \frac{9 \times 10^9 \times 64 \times (1.6 \times 10^{-19})^2}{6.04 \times 10^{-15} \times 1.6 \times 10^{-13}} MeV$$

$$U = 15.26 \, MeV$$

38. See example 4 or 5

39. See example 6

40. See example 7

41. See example 8

42. In 12.5 years half of the sample will be decayed, in the next 12.5 years again half will be decayed. Hence $\frac{1}{4}$ of the tritium will remain undecayed after 25 years.

43. $N = N_0 e^{-\lambda t}$, $\lambda = \frac{0.693}{T_{\frac{1}{2}}}$

44. a) $\lambda = \frac{0.693}{T_{\frac{1}{2}}}$ b) $N = N_0 e^{-\lambda t}$, $N_0 = 8000$, $N = 1000$ c) $\left[\frac{dN}{dt}\right]_{t=t_1} = \lambda N$

45. Total decay constant $\lambda = \lambda_\alpha + \lambda_\beta$

$\lambda_\alpha = \frac{1}{1620}$, $\lambda_\beta = \frac{1}{405}$,

Using $N = N_0 e^{-\lambda t}$, $N = \frac{N_0}{4}$

46. $R = \lambda N$, $R = 1 \mu ci = 3.7 \times 10^4$

∴ $N = 8.58 \times 10^7$

214 g of Pb contains 6.02×10^{23} atoms

∴   Mass of 8.58×10^7 atoms $= \frac{8.58 \times 10^7 \times 214}{6.02 \times 10^{23}}$

47. $N_0 = \frac{6.02 \times 10^{23} \times 200 \times 1.3 \times 10^{-12}}{12} = 1.3 \times 10^{13}$

$$\left(\frac{dN}{dt}\right)_0 = \lambda N_0 = 50s^{-1}$$

Using $\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$, $\frac{dN}{dt} = 16s^{-1}$ calculate t.

48. Initial ratio of $\frac{^{14}C}{^{12}C}$ at the time of death was, $R_0 = 1.2 \times 10^{-12}$

using $N = N_0 e^{-\lambda t}$

$$R = \frac{N(^{14}C)}{N(^{12}C)} = \frac{N_0(^{14}C)e^{-\lambda t}}{N(^{12}C)}$$

$$= R_0 e^{-\lambda t}$$

or $\quad t = \dfrac{\ln\left(\dfrac{R_0}{R}\right)}{\ln 2} T_{\frac{1}{2}}$ find t

$$R = 1.1 \times 10^{-12}, \ T_{\frac{1}{2}} = 5730 \text{ years}$$

49. We have $\lambda_1 N_1 = \lambda_2 N_2$

$$\frac{N(238)}{T_{\frac{1}{2}}(238)} = \frac{N(234)}{T_{\frac{1}{2}}(234)}$$

$$\therefore \quad T_{\frac{1}{2}}(238) = \frac{N(238)}{N(234)} T_{\frac{1}{2}}(234)$$

50. a) 13 g of N contains 6.02×10^{23} nuclei

$$\therefore \quad 1 \text{ g contains} = \frac{6.02 \times 10^{23}}{13}$$

$$1.49 \text{ mg contains} = \frac{6.02 \times 10^{23}}{13} \times 1.49 \times 10^{-6} = N_0$$

b) $R_0 = \lambda N_0$, $\lambda = \dfrac{0.693}{T_{\frac{1}{2}}} = \dfrac{0.693}{600} = 1.16 \times 10^{-3}$

c) After one hour $R = \lambda N \quad R = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$

d) $R = 1s^{-1}$, $R = \lambda N_0 e^{-\lambda t}$. Find t.

51. Try yourself

52. $\lambda N_1 = 80.5, \lambda N_2 = 36.2$

$\dfrac{N_2}{N_1} = e^{-\lambda t}$, $t = 1$ hour, find λ and use $T_{\frac{1}{2}} = \dfrac{0.693}{\lambda}$.

53. Heat produced on combustian $= 10^5 \times 8000 \times 10^3 \times 4 \text{ J} = 22 \times 10^{11} \text{J}$

Energy released per fission $= 200 \times 10^6 \times 1.6 \times 10^{-19} \text{J} = 3.2 \times 10^{-11}$

∴ Number of atoms required to produce

$32 \times 10^{11} \text{J}$ of energy $= \dfrac{32 \times 10^{17}}{3.2 \times 10^{-17}} = 10^{23}$

∴ Mass of uranium $\dfrac{235 \times 10^{23}}{6.02 \times 10^{23}} = 39.2 \text{g}$

54. See example 19
55. See example 20

56. $Q = [m(Be) - m(B)]c^2 = 11.506 \text{MeV}$

$Q = K_B + K_e + K_\upsilon$

The electrons have their maximum kinetic energy when $K_B \simeq 0$ and $K_\upsilon \simeq 0$.

57. $Q = [m(Ne) - m(Na)]c^2 = 4.376 \text{ MeV}$.

$Q = K_{Na} + K_e + K_\upsilon$ when $K_{Na} \simeq 0$ and $K_\upsilon \simeq 0$ $(K_e)_{max} = Q$.

58. $Q = [m(K) - m(Ar)]c^2 = 1.540 \text{ MeV}$

$40K + e^- = {}^{40}Ar + \upsilon$.

59. The Q-value in β^+-decay is

$Q = [m(N) - m(C^*) - 2m_e]c^2$

$m(C^*) = 12.000000 \, u + \dfrac{4.43 \text{ MeV}}{931.5 \text{ MeV/u}}$

60. a) $N_\alpha = \dfrac{232 - 208}{4} = 6$

b) Balancing the charges $90 = 82 + 2N_\alpha - N_e$ ∴ $N_e = 4$

c) $Q = [m(Th) - m(Pb) - 6m_\alpha]c^2 = 42.658974 \text{ MeV}$.

② NUCLEAR REACTIONS AND APPLICATIONS
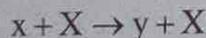
Introduction

The study of nuclear radioactive decay provides us only a little knowledge about the nucleus. This is because only few types of radioactive processes occur in nature. In these processes only few isotopes are formed and also only few excited states of nuclei can be studied. However in nuclear reaction experiments we have the freedom to select any nuclear species and any excited states of the species. Thus we obtain too many informations regarding the nucleus.
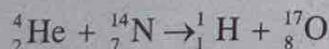
In this chapter we shall discuss some of the different nuclear reactions that can occur and we study the properties of those reactions. Two important reactions are of particular importance. They are fusion and fission reaction processes. Then we will see how these reactions are useful as sources of energy. Finally we go for applications of nuclear reactions.

Types of nuclear reactions

When a nuclei comes in contact with another nuclei, they interact with each other. In this context we say that nuclear reaction takes place. A typical nuclear reaction experiment is conducted in a laboratory is as follows. A beam of particles of type x is incident (projectile) on a target nuclei of type X. The nuclear reaction occurs. After the reaction an outgoing particle y is observed in the laboratory, leaving a residual nucleus Y. Symbolically this reaction is written as

$$x + X \rightarrow y + X$$

For example,

$$_2^4He + _7^{14}N \rightarrow _1^1H + _8^{17}O$$

Like chemical reaction, a nuclear reaction must be balanced. That is the total number of protons must be the same before and after the reaction and the total number of neutrons must be also the same before and after the reaction. In the example given above there are $(2 + 7)$ a protons on each side and 9 neutrons on each side. In this reaction there will be no beta decays. This is because in beta decays neutron can be converted into proton and vice. This will up set the balancing condition. In order

to have beta decay the time scale of interaction between projectile and the target must be of the order of 10^{-10} s. But in nuclear reactions the interaction time is only 10^{-20} s. Since there is not enough time for interaction, there will be no beta decay. It may also be noted that the protons and neutrons can be rearranged among the reacting nuclei, but their balancing should not upset.

In nuclear reactions there are only internal forces of interaction between the projectiles and the targets. As there are no external forces that come into play, nuclear reactions conserve their energy, linear momentum and angular momentum.

In most nuclear reactions we observe only the outgoing light particle y. They heavy nucleus Y usually loses all its kinetic energy, by collisions with atoms, and they stay within the target at rest.

In nuclear reactions we assume that the target particle (X) is at rest and the projectile (x) is coming with a kinetic energy k_x. The product particles share this kinetic energy plus or minus any additional energy from the rest energy difference of the initial and final nuclei.

Projectiles (x) can be either charged particles supplied by a suitable nuclear accelerators or neutrons from nuclear reactor sources. There are two types of charged particle accelerators in use. They are Cyclotron and Vandegraaf generator. Cyclotron can impart kinetic energy to charged particles of the order of 10 to 20 MeV per unit charge. The Vandegraaff accelerator can produce a potential of 25MV. The kinetic energy of the particle is about 25MeV per unit charge.

In nuclear reaction experiments we usually measure the energy of the particle y and its probability to emerge at a certain angle with certain energy.

Measuring the particle energy

We found that in nuclear reaction the kinetic energy (K_x) of the projectile (x) is shared by the product particles Y and y. If the product particles have no excited states, we can easily calculate the energy of the particle y using law of conservation of energy and momentum. If the residual nucleus is left in an excited state then the energy of the particle y is reduced by the excitation energy of Y. When the residual nucleus goes to higher and higher excited states then the energy of the particle y becomes lower and lower. This shows that measurement of the energy of the particle y gives us information about the excited state of the residual nucleus. To get the exact values energies of excited states of Y, conduct an experiment in which we measure different energies of the particle y. From this we can deduce the energies of various excited states of Y. A typical set of experimental results are plotted in a graph and shown below.
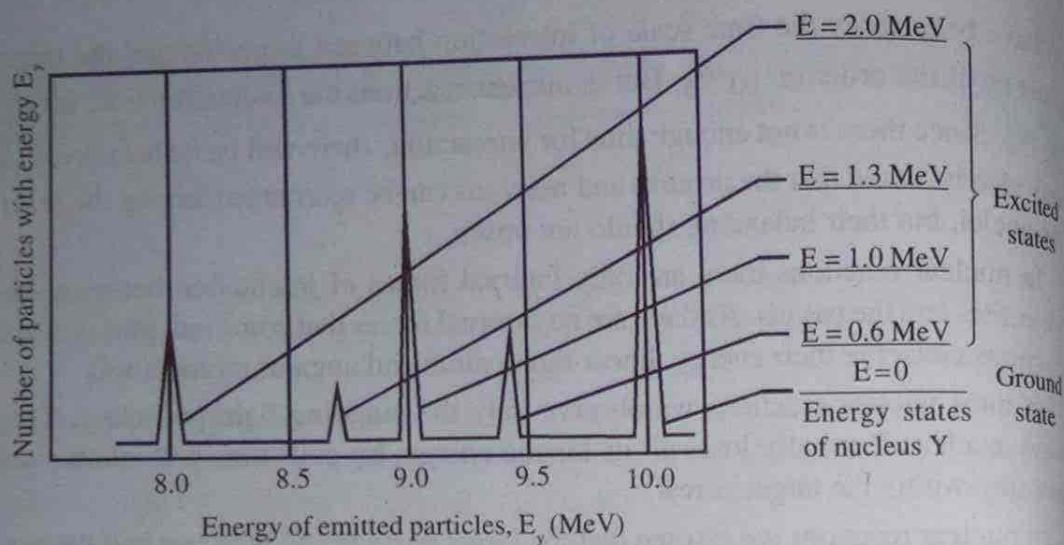
Figure 2.1: A spectrum of energies of the outgoing particles and the corresponding excited states

Each peak in the figure indicates the energy of the particle E_y. For example when $E_y = 9.0$ MeV and the corresponding energy of excited state of Y is 1.0 MeV and so on.

Measuring the reaction probability

From the figure we can see that there are different peaks having different heights. The height of the peak tells us about which is more probable. Comparing the heights we can infer about the relative probabilities. For example height of the peak corresponding to $E_y = 9$ MeV is twice the height of the peak corresponding to $E_y = 9.4$ MeV. The excited state energy corresponds to $E_y = 9$ MeV is $E_y = 1.0$ MeV and that corresponds to $E_y = 9.4$ MeV is $E_y = 0.6$ MeV. This means that in this nuclear reaction the probability of leaving Y in its excites is twice the probability of leaving Y in its first excited state ($E_y = 0.6$ MeV). To calculate the reaction probability actually we have to solve the Schrodinger equation. Unfortunately this is not possible since this is a many body problem. So what we do is starting from the experimental results and work backward together some informations regarding nuclear force.
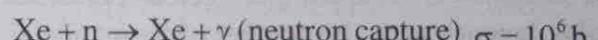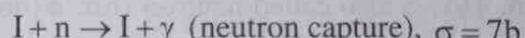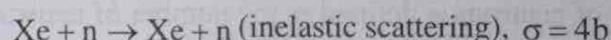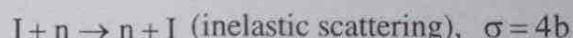
The reaction cross section

The concept of cross section plays a vital role in nuclear physics. Most of the informations about atomic nuclei are derived from nuclear reactions experiments. In

nuclear reaction experiments high energetic particles are bombarded with a stationary target nuclei. The target nuclei present a certain area, called its cross section, to the incident particles. Any incident particle that is directed at this area interacts with the target particle. Hence grater the cross section, the greater the likelihood of interaction. In other words interaction cross section (σ) is a measure of the effective size of the nucleus for a particular scattering. It can be thought of as an effective area of nucleus at right angles to the direction of motion of bombaring particles. A reaction (interaction) will occur only if the incident particle is in the area σ. Hence probability that a reaction will occur is proportional to σ. In other words cross section measures the likelihood of a particular reaction. The reaction cross section of a target particle varies with the nature of process involved and with the energy of the incident particle. It may be greater or lesser than the geometrical crossection. Cross sections have the dimensions of area and its unit is m^2. Since the size of the cross section is very very small and comparable to cross section of a nucleus another unit named barn is used.

$$1 \text{ barn} = 1b = 10^{-28} m^2.$$

The reaction cross sections of iodine and Xe non involved in reactions is given below as an example.

$I + n \rightarrow n + I$ (inelastic scattering), $\sigma = 4b$

$Xe + n \rightarrow Xe + n$ (inelastic scattering), $\sigma = 4b$

$I + n \rightarrow I + \gamma$ (neutron capture), $\sigma = 7b$

$Xe + n \rightarrow Xe + \gamma$ (neutron capture) $\sigma = 10^6 b$.

Xenon ($^{54}_{54}Xe$) and iodine ($_{53}I$) are isotopes of neighbouring elements. In inelastic scattering both have same cross sections ($\sigma = 4b$). But in neutron capture reactions Xe and I have very different cross sections telling us about the unusual properties of the nucleus Xenon.

Expression for cross section

Suppose a beam of partiacles is incident on a thin target of area S which contains a total of N nuclei. The effective area of each nucleus is the cross section σ, and so the total effective area of all the nuclei in the target is σN. The fraction of the target area that this represents is $\dfrac{\sigma N}{S}$. This fraction is the probability for the reaction to occur.

Suppose the incident particles strike the target at a rate of I_0 particles per second. The number of particles per second is called beam intensity. After reaction the outgoing particles y are emitted at a rate of R per second. This is also the rate at which the product nucleus is formed. Then the reaction probability can also be expressed as the rate of y divided by the rate of x i.e.,

The reaction probability $= \dfrac{R}{I_0}$

The reaction probability already obtained $= \dfrac{\sigma N}{S}$

Equating the two, we get

$$\frac{R}{I_0} = \frac{\sigma N}{S}$$

or
$$R = \frac{\sigma N}{S} I_0 \qquad\qquad ..... (1)$$

This is the relation between the rate of emission of y (R) and the reaction cross section (σ).

In a reactor, the beam intensity of neutrons is defined as the number of neutrons cross unit area perpendicular to the beam. This is also called neutron flux ϕ (neutrons per cm^2 per second). The cross section is σ (It is cm^2 per nucleus per incident neutron). The rate of emission of y also depends upon the number of target nuclei. If M is the molar mass of the target. This means that M grams containing one Avogadro number (N_A) of atoms.

$\therefore$ 1 gram contains $\dfrac{N_A}{M}$ atoms.

If m (gram) is the mass of the target. Then the number of atoms in the target

$$N = \frac{m N_A}{M}$$

Equation (1) can be rewritten as $R = \sigma N \left(\dfrac{I_0}{S} \right)$

$\dfrac{I_0}{S}$ is the neutron intensity (ϕ)

$$R = \sigma N \phi$$

substituting for N $\quad R = \sigma \phi \dfrac{m N_A}{M}$ $\qquad\qquad$ ..... (2)

Example 1

A beam of $20.0\,\mu A$ of protons is incident on $2.0\ cm^2$ of a target of ^{107}Ag of thickness $4.5\ \mu m$ producing the reaction.

$$p + {}^{107}Ag \rightarrow {}^{105}cd + 3n$$

Neutrons are observed at a rate of 8.5×10^6 per second. What is the cross section for this reaction at this proton energy. Density of $Ag = 10.5\ gcm^{-3}$.

Solution

We know that, current $= \dfrac{\text{Total charge}}{\text{time}}$

$$20\ \mu A = \dfrac{Ne}{t}$$

$\therefore$ Number of protons incident per second

$$\dfrac{N}{t} = \dfrac{20\mu A}{e} = \dfrac{20 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{14}$$

i.e., Intensity of the proton beam

$$I_0 = 1.25 \times 10^{14}\ \text{protons s}^{-1}$$

Using $\qquad R = \dfrac{\sigma N I_0}{S}$

But $\qquad \dfrac{N}{S} = \dfrac{(m N_A / M)}{S} = \dfrac{V \rho N_A}{S M}$

$$\dfrac{N}{S} = \left(\dfrac{V}{S}\right) \dfrac{\rho N_A}{M} = \dfrac{t\ \rho N_A}{M}$$

Where t is the thickness of the target

$$\therefore \quad R = \frac{\sigma I_0\, t\, \rho N_A}{M}$$

or

$$\sigma = \frac{RM}{I_0 t \rho N_A}$$

$$R = \frac{1}{3}(8.5 \times 10^6)\ s^{-1} \quad \text{(given)}$$

$$M = 107 \text{gmole}^{-1},\ \ I_0 = 1.25 \times 10^{14}$$

$$\rho = 10.5 \text{g cm}^{-3},\quad t = 4.5 \times 10^{-4} \text{cm}$$

$$N_A = 6.022 \times 10^{23}\ \text{atoms / mole}$$

$$\therefore \quad \sigma = \frac{8.5 \times 10^6}{3} \times \frac{107}{1.25 \times 10^{14} \times 4.5 \times 10^{-4} \times 10.5 \times 6.022 \times 10^{23}}$$

$$\sigma = 8.523 \times 10^{-28}\ cm^2 = 8.523 \text{barn}$$

Example 2

A beam of alpha particles is incident on a target of ^{63}Cu, resulting in the reaction $\alpha + {}^{63}Cu \rightarrow {}^{66}Ga + n$. Assume that cross section for the particular alpha energy to be 1.25b. The target is in the form of a foil 2.5μm thick. The beam has a circular cross section of diameter 0.50 cm and a current of 7.5μA. Find the rate of neutron emission.

Density of copper = 8.96 gcm^{-3}

Solution

$$\sigma = 1.25b = 1.25 \times 10^{-24} cm^2,\quad t = 2.5 \times 10^{-4} cm$$

$$d = 0.50 cm,\quad \text{current} = 7.5 \mu A$$

$$M = 63g \text{ and } \rho = 8.96\ \text{gcm}^{-3}$$

Using $\quad R = \dfrac{\sigma I_0 N}{S}$

$$\text{Current} = \frac{\text{Total charge}}{\text{time}} = \frac{N.2e}{t}$$

$$\therefore \quad \frac{N}{t} = I_0 = \frac{\text{Current}}{2e} = \frac{7.5 \times 10^{-6}}{2 \times 1.6 \times 10^{-19}}$$

$$I_0 = 2.344 \times 10^{13} \text{ particles s}^{-1}$$

$$\frac{N}{S} = \frac{mN_A}{SM} = \frac{V\rho N_A}{\pi r^2 M} = \frac{t\rho N_A}{M}$$

$$\frac{N}{S} = \frac{2.5 \times 10^{-4} \times 8.96 \times 6.022 \times 10^{23}}{63}$$

$$\frac{N}{S} = 2.141 \times 10^{19}$$

$$\therefore \quad R = 1.25 \times 10^{-24} \times 2.141 \times 10^{19} \times 2.344 \times 10^{13}$$

$$R = 6.273 \times 10^8 \text{ neutrons per second.}$$

Radioisotope production in nuclear reactions

Nuclear reactions are also used to produce radioisotopes. In this process of reaction a stable nonradioactive isotope X is irradiated with a particle x to form the radioactive isotope Y. The outgoing particle is y. Here the outgoing particle is not our interest so we don't observe them. Our interest is to observe the number of radio active isotopes formed that remains at rest within the target after reaction. Since the particle Y is radioactive it will undergo decay.

Here our aim is to calculate the activity of the isotope Y for a certain time t. Let R be the constant rate at which Y is produced. This is related to the intensity (I_0) of the particle x (see equation 1). In a time interval dt, the number of Y nuclei produced is Rdt. Since Y is radioactive it decays. In time dt the number of Y nuclei decayed is λNdt, where λ is the decay constant and N is the number of Y nuclei present.

Therefore the net change in the number (dN) of Y nuclei is

$$dN = Rdt - \lambda Ndt$$

or

$$\frac{dN}{dt} = R - \lambda N$$

$$\frac{dN}{dt} + \lambda N = R$$

Multiplying throughout by the integrating factor $e^{\lambda t}$, we get

$$e^{\lambda t}\frac{dN}{dt} + e^{\lambda t}\lambda N = Re^{\lambda t}$$

or

$$\frac{d}{dt}(Ne^{\lambda t}) = Re^{\lambda t}$$

Integrating we get

$$Ne^{\lambda t} = \frac{Re^{\lambda t}}{\lambda} + c \qquad \qquad \text{..... (4)}$$

Where c is the constant of integration

when $t = 0$, $N = 0$

$$\therefore \qquad 0 = \frac{R}{\lambda} + c$$

i.e., $$c = -\frac{R}{\lambda}$$

Put this in equation 4, we get

$$Ne^{\lambda t} = \frac{Re^{\lambda t}}{\lambda} - \frac{R}{\lambda}$$

$$N = \frac{R}{\lambda} - \frac{R}{\lambda}e^{-\lambda t}$$

$$N = \frac{R}{\lambda}(1 - e^{-\lambda t}) \qquad \qquad \text{..... (5)}$$

or $$\lambda N = R(1 - e^{-\lambda t})$$

λN is called activity denoted by $a(t)$

Thus, activity

$$a(t) = R(1 - e^{-\lambda t}) \qquad \qquad \text{..... (6)}$$

When t = 0, a(t) = 0. This shows that at t = 0, there is no radioactive isotope of y hence activity is zero. Equation 6 tell us that as the irradiation time (t) passes, the activity a(t) of the isotope increases exponentially. See figure shown below.


Irradiation time (units of half-life)
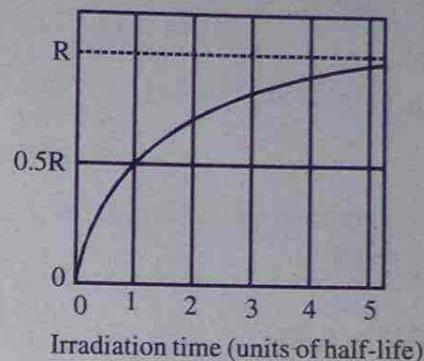
Figure 2.2:   Formation of activity in a nuclear reaction

For large irradiation times $t \gg T_{1/2}$,

$$a(t) = R \qquad\qquad ..... (7)$$

When $t \ll T_{1/2}$, $\lambda t = \dfrac{0.693}{T_{1/2}} t$ is very very small. Thus $e^{-\lambda t} \simeq 1 - \lambda t$

$$\therefore \qquad a(t) = R(1 - e^{-\lambda t}) = R(1 - (1 - \lambda t))$$

$$a(t) = R\lambda t \qquad\qquad ..... (8)$$

Example 3

A radioactive isotope o half life $T_{1/2}$ is produced in a nuclear reaction. What fraction of the maximum possible activity is produced in an irradiation time of

a) $T_{1/2}$ b) $2 T_{1/2}$ c) $4 T_{1/2}$

Solution

Maximum activity of the isotope

$$a_{max} = R$$

The fraction of maximum activity,

$$f = \frac{a(t)}{R} = \frac{R(1 - e^{-\lambda t})}{R} = 1 - e^{-\lambda t}$$

a) $f = 1 - e^{-\lambda t} = 1 - e^{\frac{-0.693}{T_{1/2}} T_{1/2}} = 1 - e^{-0.693}$

$f = 1 - 0.50 = 0.50$

b) $f = 1 - e^{\frac{-0.693}{T_{1/2}} \cdot 2T_{1/2}} = 1 - e^{-0.693 \times 2}$

$f = 1 - 0.5^2 = 0.75$

c) $f = 1 - e^{\frac{-0.693}{T_{1/2}} \cdot 4T_{1/2}} = 1 - e^{-0.693 \times 4}$

$f = 1 - (0.5)^4 = 1 - 0.0625 = 0.9375$

Note: $e^{-0.693} \simeq 0.5$

Example 4

Neutron capture in sodium occurs with a cross section of 0.53b and leads to radioactive ^{24}Na $(T_{1/2} = 15h)$. What is the activity that results when 1.00 µg of Na is placed in a neutron flux 2.5×10^{13} neutron $cm^{-2}s^{-1}$ for 4.00 hours.

Solution

$$\sigma = 0.536\,b = 0.53 \times 10^{-24}\,cm^2$$

$$T_{1/2} = 15h$$

$$m = 1.00\mu g = 1.00 \times 10^{-6}\,g$$

$$\phi = 2.5 \times 10^{13}\ \text{neutrons}\ cm^{-2}s^{-1}$$

$$t = 4.00h \quad \text{and} \quad M = 23$$

Using $R = \dfrac{\sigma\phi m N_A}{M}$

$$R = \frac{0.53 \times 10^{-24} \times 2.5 \times 10^{13} \times 10^{-6} \times 6.02 \times 10^{23}}{23}$$

$$R = 3.47 \times 10^5\ s^{-1}$$

∴   Activity, $a(t) = R\,(1 - e^{-\lambda t})$

$$a(t) = 3.47 \times 10^5 \left(1 - e^{-\frac{0.693}{15} \times 4}\right)$$

$$a(t) = 3.47 \times 10^5 \ (1 - 0.831)$$

$$a(t) = 3.47 \times 10^5 \ \times 0.169 = 5.86 \times 10^4 \, s^{-1}$$

Low energy reaction kinematics

Consider a projectile x moving with momentum $\vec{p}_x$ and kinetic energy K_x. The target is at rest and the reaction products (y and Y) have momenta $\vec{p}_y$ and $\vec{P}_y$ and kinetic energies K_y and K_Y. The particles y and Y are emitted at angles θ_y and θ_Y with respect to the initial direction of the projectile. This is illustrated in the figure given below. We assume that the product nucleus Y is not observed in the labora-tory. If it is a heavy nucleus it moves with slow speed or generally comes to rest within the target. One more assumption that we make is, the velocities of the nuclear particles are sufficiently small that we can use non-relativistic kinematics.
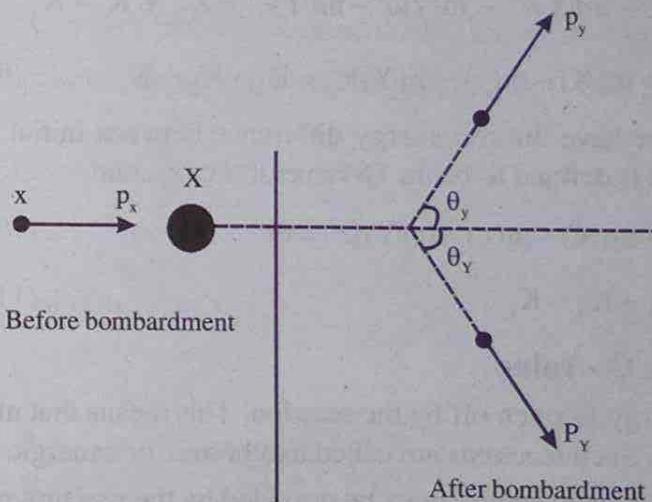

Figure 2.3

Our aim here is to find out the Q-value of the reaction.

Using law the conservation of energy:

Energy before reaction = Energy after reaction

i.e., $m_x c^2 + K_x + m_X c^2 = m_y c^2 + K_y + m_Y c^2 + K_Y$

All masses are nuclear masses.

We found that in nuclear reactions number of protons are balanced.

i.e., $z_x + Z_X = z_y + Z_Y$

We can therefore add equal number of electrons on both sides. We get

$$m_x c^2 + z_x m_e c^2 + K_x + m_X c^2 + Z_X m_e c^2$$

$$= m_y c^2 + z_y m_e c^2 + K_y + m_Y c^2 + Z_Y m_e c^2 + K_Y$$

i.e, $[m_x + z_x m_e] c^2 + K_x + [m_X + Z_X m_e] c^2$

$$= [m_y + z_y m_e] c^2 + K_y + [m_Y + z_Y m_e] c^2 + K_Y$$

The terms inside the square brackets are atomic masses of respective nuclei.

i.e, $m(x)c^2 + K_x + m(x)c^2 = m(y)c^2 + K_y + m(Y)c^2 + K_Y$

or $m(x)c^2 + m(X)c^2 - m(y)c^2 - m(Y)c^2 = K_Y + K_y - K_x$

$$[m(x) + m(X) - m(y) - m(Y)]c^2 = K_y + K_Y - K_x \quad \text{..... (9)}$$

On the L.H.S we have the rest energy difference between initial particles and final particles. This is defined to be the Q-value of the reaction.

$$[m(x) + m(X) - m(y) - m(Y)]c^2 = Q \quad \text{..... (10)}$$

Thus $Q = K_y + K_Y - K_x$ ..... (11)

Discussion on the Q - value

If $Q > 0$, the energy is given off by the reaction. This means that nuclear energy has been converted. Such reactions are called exothermic or exoergic reactions.

If $Q < 0$, enough kinetic energy must be provided by the reacting particles. This means that these reactions require input energy in the form of the kinetic energy of x. Such reactions are called endothermic or endoergic reactions.

In endoergic reaction, we must supply enough kinetic energy to provide the additional rest energy of the reaction products. There is thus some minimum or threshold kinetic energy of x below which the reaction will not take place. This threshold kinetic energy not only must supply the additional rest energy but also some kinetic

energy to the products. This is because after reaction either one product particle or two product particles are in motion. For this motion they require kinetic energy. If both particles are at rest after reaction this would violate law of conservation of momentum. This problem can easily analyse in the centre of mass reference frame.

Lab frame and centre of mass frame - A review

Consider a particle of mass $m(x)$ moving with speed u_x approaches a particle of mass $m(X)$ at rest. The frame of reference with origin at $m(X)$ defines the lab frame.
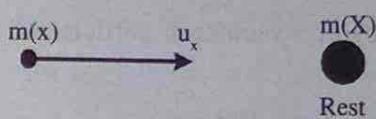
Figure 2.4

The centre of mass of $m(x)$ and $m(X)$ is defined to be

$$R_{cm} = \frac{m(x)\, r_x + m(X)\, r_X}{m(x) + m(X)}$$

Where r_x and r_X are the position vectors of $m(x)$ and $m(X)$ respectively.

Differentiating the above equation with respect to time

$$\frac{dR_{cm}}{dt} = \frac{m(x)\dfrac{dr_x}{dt} + m(x)\dfrac{dr_X}{dt}}{m(x) + m(X)}$$

i.e.
$$V_{cm} = \frac{m(x)u_x + m(X)u_X}{m(x) + m(X)}$$

But $u_X = 0$ $\therefore$ $V_{cm} = \dfrac{m(x)u_x}{m(x) + m(X)}$

In the lab frame kinetic energy is that of the incident particle only.

i.e,
$$K_{lab} = \frac{1}{2}m(x)v_x^2$$

Centre of mass frame

Consider two particles of masses $m(x)$ and $m(X)$ moving with velocities u'_x and u'_X. Here the origin of the co-ordinate system is at the centre of mass we have

$$R'_{cm} = \frac{m(x)r'_x + m(X) r'_X}{m(x) + m(X)} = 0$$

i.e, $$m(x)r'_x = -m(X) r'_X$$

Where r'_x and r'_X are the position vectors of particles of masses $m(x)$ and $m(x)$ respectively.

Differentiating with respect to time, We get

$$m(x)\frac{dr'_x}{dt} = -m(X)\frac{d r'_X}{dt}$$

i.e. $$m(x)u'_x = -m(X)u'_X$$

This means that in C.M frame the direction of the two particles will be opposite to each other.
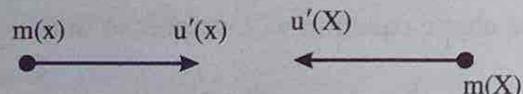
Figure 2.5

Relation between velocities in lab frame and that in C.M frame

From the figure we have

$$\vec{r}_x = \vec{r}'_x + \vec{v}_{cm}t$$

Differentiating with respect to time, we get

$$\vec{u}_x = \vec{u}'_x + v_{cm}$$

$\therefore$ $$u'_x = u_x - v_{cm}$$

and $$u'_x = u_x - v_{cm} \text{ But } u_x = 0$$

$\therefore$ $$u'_x = -v_{cm}$$
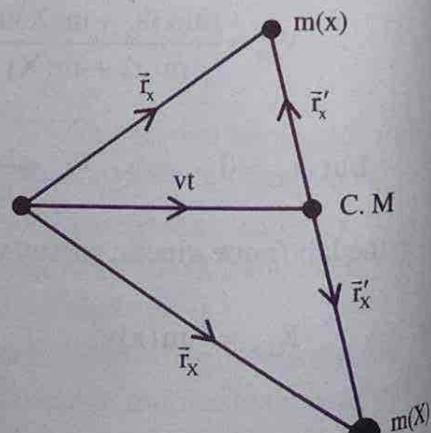
Figure 2.6

or $\qquad u'_x = u_x - \dfrac{m(x)u_x}{m(x)+m(X)}$

$\therefore \qquad u'_x = \dfrac{m(X)u_x}{m(x)+m(X)} \qquad\qquad\qquad ..... (12)$

and $\qquad u'_X = -\dfrac{m(x)u_x}{m(x)+m(X)} \qquad\qquad\qquad ..... (13)$

When you go from Lab frame to C.M frame u_x [the speed of $m(x)$] must be replaced by u'_x and the speed of $u_x (u_X = 0)$ must be replaced by u'_X.

Expression for threshold kinetic energy

In the centre of mass reference frame the projectile x of mass $m(x)$ is moving with a speed u'_x and the target particle (X) moving with a spead u'_x in opposite directions. After nuclear reaction the product particles y and Y would be at rest. Since we find the threshold kinetic energy.


Before bombardment in C.M frame

After bombardment in C.M frame
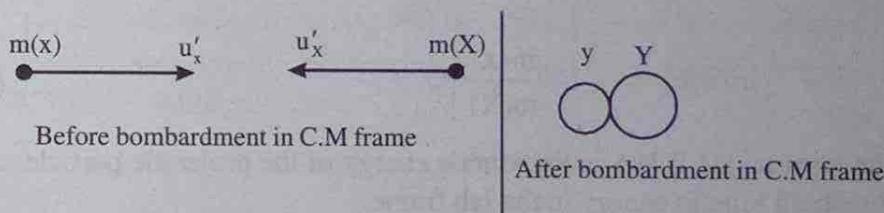
Figure 2.7

Using law of conservation of energy, we have

Total energy before reaction = Total energy after reaction

$$m(x)c^2 + \frac{1}{2}m(x)u'^2_x + m(X)c^2 + \frac{1}{2}m(X)u'^2_X = m(y)c^2 + m(Y)c^2$$

or $\quad \dfrac{1}{2}m(x)u'^2_x + \dfrac{1}{2}m(X)u'^2_X = m(y)c^2 + m(Y)c^2 - m(x)c^2 - m(X)c^2$

Substituting for u'_x from eq. (12) and u'_x from eq. (13) in the above, we get

$$\frac{1}{2}m(x)\left[\frac{m(X)u_x}{m(x)+m(X)}\right]^2 + \frac{1}{2}m(X)\left[-\frac{m(x)u_x}{m(x)+m(X)}\right]^2$$

$$= m(y)c^2 + m(Y)c^2 - m(x)c^2 - m(X)c^2$$

$$\frac{\frac{1}{2}m(x)m^2(X)u_x^2}{[m(x)+m(X)]^2} + \frac{\frac{1}{2}m(X)m^2(x)u_x^2}{[m(x)+m(X)]^2} = -[m(x)+m(X)-m(y)-m(Y)]c^2$$

$$\frac{1}{2}m(x)u_x^2 \; m(X)\frac{[m(X)+m(x)]}{[m(x)+m(X)]^2} = -Q$$

On the R.H.S we have the Q-value of the reaction

$$\therefore \qquad \frac{1}{2}m(x)u_x^2 \frac{m(X)}{m(x)+m(X)} = -Q$$

or $\qquad \dfrac{1}{2}m(x)u_x^2 = -\dfrac{m(x)+m(X)}{m(X)}Q$

$$\frac{1}{2}m(x)u_x^2 = -\left[1+\frac{m(x)}{m(X)}\right]Q \qquad\qquad \text{..... (14)}$$

The term on the L.H.S is the kinetic energy of the projectile particle x, which the threshold kinetic energy in the lab frame.

i.e. $\qquad K_{Th} = -\left[1+\dfrac{m(x)}{m(X)}\right]Q \qquad\qquad$ ..... (15)

This is the expression for the threshold kinetic energy.

Note: While deriving the expression for K_{Th}, we used law of conservation of energy. The rest energy is mc^2, which is relativistic but kinetic energy is non-relativistic. This is admissible since $u_x \ll c$.

Example 5

Find the Q-value of the reaction

$$^3He + {}^{40}Ar \rightarrow {}^{41}K + {}^2H$$

Given: m(He) = 3.016029 u, m(Ar) = 39.962383 u

m(K) = 40.961826 u and m(H) = 2.014102 u.

Solution

Q- value of the reaction is :

$$Q = [m(He) + m(Ar) - m(K) - m(H)]c^2$$

$$Q = [3.016029\,u + 39.962383\,u - 40.961826\,u - 2.014102\,u]c^2$$

$$Q = 0.2484\,u\,c^2$$

Using $1u = 931.5\dfrac{MeV}{c^2}$

$$Q = 0.2484 \times 931.5 \text{ MeV}$$

$$Q = 2.313846 \text{ MeV}$$

Example 6

In the reaction $^2H + \,^3He \rightarrow p + \,^4He$, deutrons of energy 5.000 MeV are incident on ^3He at rest. Both the proton and the alpha particle are observed to travel along the same direction as the incident deutron. Find the sum of kinetic energies of the proton and the alpha particle.

Given: $m(^2H) = 2.014102\,u, m(^3He) = 3.016029\,u$

$m(^1H) = 1.007825\,u$ and $m(^4He) = 4.002603\,u$

Solution

The Q-value of the reaction is

$$Q = \left[m(^2H) + n(^3He) - m(^1H) - m(^4He) \right]c^2$$

$$Q = \left[2.014102u + 3.016029\,u - 1.007825\,u - 4.002603\,u \right]c^2$$

$$Q = 0.019703\,u\,c^2$$

$$Q = 0.019703 \times 931.5\,MeV$$

$$Q = 18.353 \text{ MeV}$$

But $Q = K_y + K_Y - K_x$ in general

Here $\quad Q = K_p + K_{^4He} - K_{^2H}$

$$18.353 = K_p + K_{^4He} - 5.000$$

$$K_p + K_{^4He} = 23.353 \text{ MeV}$$

Example 7

What is the Q-value of the reaction $p + {}^4He \rightarrow {}^2H + {}^3He$

a) What is the threshold energy for protons incident on ${}^4He$ at rest.

b) What is the threshold energy if ${}^4He$ are incident on proton at rest m(H) $=1.007825u$, $m({}^4He) = 4.002603u$, $m({}^2H) = 2.014102u$ and $m({}^3He) = 3.016029$ u.

Solution

$$Q = \left[m({}^1H) + m({}^4He) - m({}^2H) - m({}^3He) \right] c^2$$

$$Q = \left[1.007825u + 4.002603u - 2.014102u - 3.016029u \right] c^2$$

$$Q = -0.019703uc^2$$

$$Q = -0.019703 \times 931.5 \text{MeV}$$

$$Q = -18.353$$

a) Threshold kinetic energy

$$K_{Th} = -Q \left[1 + \frac{m(x)}{m(X)} \right]$$

$$K_{Th} = -Q \left[1 + \frac{m({}^1H)}{m({}^1He)} \right]$$

$$K_{Th} = 18.353 \left[1 + \frac{1.007825}{4.002603} \right]$$

$$K_{Th} = 22.974 \text{ MeV}$$

b) $K_{Th} = -Q \left[1 + \dfrac{m(^4He)}{m(^1H)} \right]$

$K_{Th} = --18.353 \left[1 + \dfrac{4.002603}{1.007825} \right]$
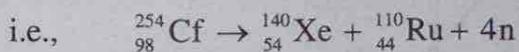
$K_{Th} = 91.242 \ MeV$

Nuclear Fission

Nuclear fission is the phenomenon of splitting a heavy nucleus (A>230) into two lighter nuclei with the release of energy. These are of two types. (1) Spontaneous nuclear fission and (2) induced nuclear fission.
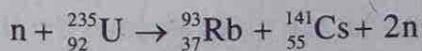
Spontaneous nuclear fission

By a suitable selection of a projectile and a target we can conduct collisions in laboratories to produce a heavy nucleus which is radioactive. For example the massive nucleus californium $^{254}_{98}Cf$ can be produced in accelerators by collision. This heavy nucleus is radio active, decaying with a half life 60.5 days. The Californium decays into xenon and ruthenium and with the release of four neutrons.

i.e., $^{254}_{98}Cf \rightarrow {}^{140}_{54}Xe + {}^{110}_{44}Ru + 4n$

If we calculate the Q-value for positive and negative beta decay, they are found to be negative. But the Q-value of alpha decay is positive. Thus alpha is energetically possible for californuim. But this alpha particles cannot overcome the high Coulomb barrier of californium. So alpha decay is also not possible. In effect californium does not undergo nuclear decays (α or $\beta^{\pm}$) but disintegrate into smaller nuclei. This mode of decay is known as nuclear fission.

When a heavy nucleus is bombarded with neutrons, nucleus becomes excited. This excited nucleus is unstable and disintegrates into two lighter nuclei. This is known as induced nuclear fission. For example

$n + {}^{235}_{92}U \rightarrow {}^{93}_{37}Rb + {}^{141}_{55}Cs + 2n$

Note: The Californium nucleus was particularly selected owing to some special interest. This is because it is also produced in supernova explosions and knowledge of its properties provides a key to understanding the formation of elements in stars.

Mechanism of fission

A nucleus consists of protons and neutrons moving about under the mutual attraction of their nuclear forces and Coulombian repulsive force. As a result of these interactions many nuclei assume spherical in shape and behave like liquid drops according to liquid drop model. When this nucleus experiences some external force (arising from external energy) the equilibrium spherical shape will be distorted and becomes ellipsoidal in shape. Such a nucleus undergoes rapid oscillations about their equilibrium shape. In other words nucleus behaves like a stretched spring when released. see figure (2.8).

Other nuclei which are already in ellipsoidal shape at equilibrium, when such a nucleus experiences an external force (arising from external energy) it undergoes rapid oscillations and finally comes back to their ellipsoidal shape. If the external force is large nucleus may not return to its equilibrium but instead may split into two. See figure (2.9).

For example, consider the collision between a neutron and a heavy nucleus $^{236}_{92}U$. They combine to form a compound nucleus which is

Figure 2.8: The energy of a nucleus with a spherical equilibrium shape increases as the distortion increases
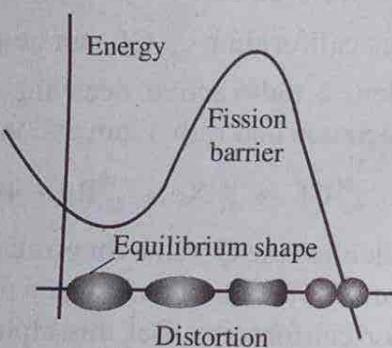
Figure 2.9: The energy of a nucleus with a nonspherical equilibrium shape. If enough energy is added, the nucleus can tunnel through the fission barrier and split into two pieces

highly energetic. Its extra energy is partly the kinetic energy of the neutron but largely the added binding energy of the incident neutron. This energy appears to initiate a series of rapid oscillations in the drop which at times assumes the shape B shown in the figure below. The restoring force of the nucleus arises from the short range inter nucleon forces. If the oscillations become so violent that the stage D is reached and each half is now positively charged, the final fission into stage E is inevitable. Thus this is a threshold energy or a critical energy required to produce stage D after which the nucleus cannot return to A, because of Coulombian repulsion of the two parts.

Spherical compound
nucleus e.g, $_{92}^{236}$U   Ellipsoidal   Narrowing   Critical state   Separation

A   B   C   D   E

Figure 2.10: Mechanism of fission in liquid-drop model of nucleus

The critical energy which must be supplied with the neutron, is best illustrated in figure below which is a potential energy diagram. In this diagram we see how the energy E_{crit} must be added to the system to enable the energy of the nucleus to become greater than the stability barrier energy E_b. Once the maximum barrier height has been overcome the system descends to the state of lowest potential energy and the fragments separate. When the mass of the compound nucleus is greater than the masses of the total fission fragments, fission is possible and the mass difference is released as energy according to Einstein's relation $\Delta E = \Delta mc^2$.
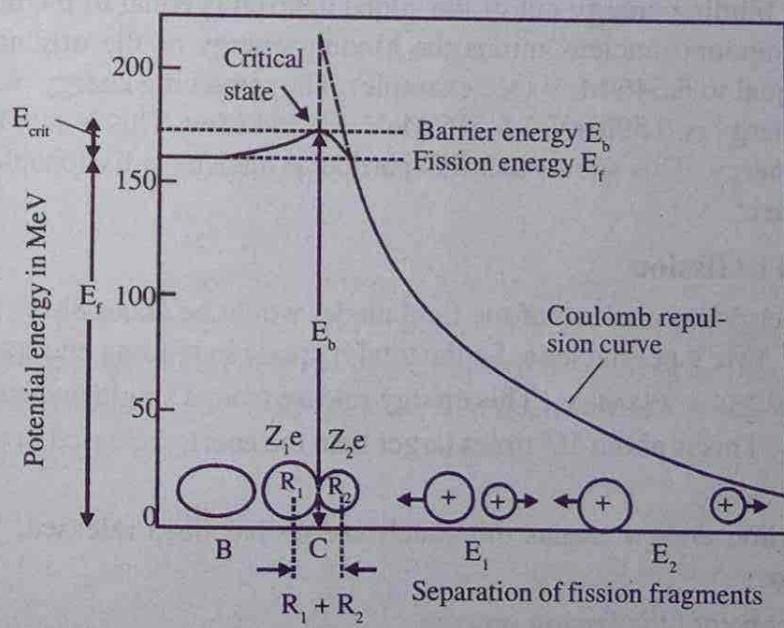

Figure 2.11: Potential energy curve for fission

Expression for critical energy

The value of critical deformation energy E_{cri} was first calculated by Bohr and wheeler based on the liquid drop model. They found

$$E_{cri} = 0.89A^{2/3} - 0.02\frac{Z(Z-1)}{A^{1/2}} \text{ MeV} \qquad \qquad .....(16)$$

For example, consider

$^{235}_{92}U$ combines with a neutron forming $^{236}_{92}U$. We calculate its critical energy.

$$E_{cri} = 0.89A^{2/3} - 0.02\frac{Z(Z-1)}{A^{1/3}} \text{ MeV}$$

$$E_{cri} \text{ of } {}^{236}_{92}U = 0.89 \times (236)^{2/3} - 0.02 \times \frac{92 \times (92-1)}{(236)^{1/3}} \text{ MeV}$$

$$= 0.89 \times 38.19 - 29.10$$

$$= 33.99 - 29.10$$

$$= 6.89 \text{ MeV}$$
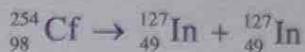
This energy must be added as the kinetic energy and binding energy of the incident neutron. The binding energy out of the added neutron is equal to the binding energy of 236 compound nucleus minus the binding energy of the original 235 nucleus. This is equal to 6.546 MeV (see example). The remaining energy required to attain critical energy is 6.89MeV – 6.546 MeV = 0.344 Mev. This is supplied as neutrons kinetic energy. This shows that this particular nucleus is fissionable with low energy neutrons.

Energy released in fission

The binding energy per nucleon of the final nuclei would be about 8MeV. Thus there is increase of 1MeV per nucleon. So the total increase in binding energy of the nucleus is about $1 \times 254 = 254$MeV. This energy release from a single nucleus is an enormous quantity. This is about 10^8 times larger than the energy released in chemical processes.

Increase in binding energy means this much energy has been released. Where does this energy go?

To clarify this consider the fission process

$$^{254}_{98}\text{Cf} \rightarrow \,^{127}_{49}\text{In} + \,^{127}_{49}\text{In}$$

when they are about to split two positively charged indium nuclei, just touching at their surfaces

The radius of each nuclei is

Figure 2.12

$$r = r_0 \, A^{1/3}$$

$$r = 1.2 \, (127)^{1/3} = 6.03 \text{fm}$$

∴ The separation between the two nuclei $= 2r = 12.06$ fm

The Coulombian repulsion between them is

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 (2r)} = \frac{(Z_1 e)(Z_2 e)}{4\pi\varepsilon_0 \cdot 2r}$$

$$U = \frac{9 \times 10^9 \, (49\,e)(49\,e)}{12.06 \times 10^{-15}} \text{ J}$$

$$U = \frac{9 \times 10^9 \times (49)^2 (1.6 \times 10^{-19})^2}{12.06 \times 10^{-15}}$$

$$U = \frac{9 \times 10^9 \, (49)^2 \times (1.6 \times 10^{-19})^2}{12.06 \times 10^{-15} \times 1.6 \times 10^{-13}} \text{ MeV}$$
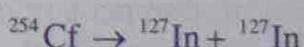
$$U = \frac{34574.4 \times 10^{-1}}{12.06}$$

$$U = 286.68 \text{ MeV}$$

This is very much close to our estimate of the binding energy released. These two charged objects repel one another, so that Coulomb energy quickly becomes kinetic energy. This energy is given to the fission products, the decay products (betas and gammas) and other particles like neutrons.

Example 8

Find the Q-value of the fission decay

$$^{254}\text{Cf} \rightarrow \,^{127}\text{In} + \,^{127}\text{In}$$

m(In) = 126.917353 u   and m (Cf) = 254.087323 u
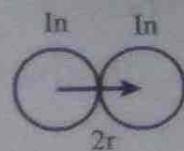
Solution

$$Q = \left[m(Cf) - 2m(In)\right]c^2$$

$$Q = \left[254.087323u - 2 \times 126.917353u\right]c^2$$

$$Q = 0.252617uc^2$$

$$Q = 0.252617 \times 931.5 MeV = 235.312 \, MeV$$

Example 9

Find the energy released in the fission of 1.00kg of uranium that has been enriched to 3.0% in the radioisotope of ^{235}U. Each fission releases about 200 MeV.

Solution

The mass of isotope that contains in 1.00kg $= 1.00 \times \dfrac{30}{100} = 0.03 kg = 30 g$

∴   The number of atoms in

$$^{235}U \text{ isotope} = \frac{m \, N_A}{M} = \frac{30 \times 6.022 \times 10^{23}}{235}$$

$$= 7.689 \times 10^{22} \, atoms$$

Since fission releases about 200 MeV, the total energy released,

$$E = 7.689 \times 10^{22} \times 200$$

$$E = 15.378 \times 10^{24} \, MeV$$
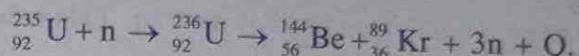
or $$E = 15.378 \times 10^{24} \times 1.6 \times 10^{-13} J$$

$$E = 24.6 \times 10^{11} = 2.46 \times 10^{12} J$$

Induced fission

We found that radio active decay of californium is an example of spontaneous fission. There are other examples of fissionable nuclei that may occur naturally or are produced artificially. These nuclei can be made to fission by the addition of some energy, which might be in the form of absorption of photons or neutrons. In these cases the energy input is very small compared with the energy released in the fission process.

An example of such fission process is

$$^{235}_{92}U + n \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Be + {}^{89}_{36}Kr + 3n + Q.$$

The Q-value in this reaction is about 200 MeV.

In a bulk sample of uranium each of the neutrons emitted in fission can be absorbed by another ^{235}U and thus induce another fission process, resulting in the emission of still more neutrons. This will be followed by more fissions and so forth. As long as the average number of neutrons available to produce new fissions is greater than 1per nucleon, the number of fissions grows with time. the avalanche or chain reaction of fission occurs each with a release of about 200 MeV energy. This occurs in a rapid and uncontrolled conditions as in nuclear weapons. The energy output can be taken out under slower and carefully controlled conditions as in a nuclear reactor.

Electrical power from fission

Nuclear energy originates from the splitting of uranium nuclei by a process called nuclear fission. The thermal energy so produced is used to boil water producing steam. This steam is used by a turbine generator to generate electricity. But there is a problem, ordinary uranium by itself cannot undergo fission even if it is bombarded by a neutron. There are mainly three reasons for this. They are called (1) enrichment (2) moderation and (3) control.

Enrichment

Natural uranium consists of 0.7% of ^{235}U and 99.3% of ^{238}U. ^{235}U is fissionable but ^{238}U is not fissional. To produce fission in the ^{235}U, we require one neutron and one fission produces two neutrons. If there are two neutrons absorbed by ^{238}U, no further fission occurs. If by chance one neutron is free among the two neutrons produced which can initiate another fission of ^{235}U. So we cannot expect a sustained production of energy. To maintain a steady energy production from fission reactions, we should have one free neutron from each fission produce another fission. To overcome this problem it is necessary to use enriched uranium, which contains 3-5% of ^{235}U. Enrichment is a difficult process because ^{235}U and ^{238}U are chemically identical. By forcing gaseous uranium through a porus barrier in which the more massive ^{238}U atoms diffuse more slowly there by we get more ^{235}U in the output.

Moderation

The neutrons released in fission have high kinetic energies of the order of a few

MeV. Such neutrons cannot trigger the fission of ^{235}U further. This is because the fission cross section generally decreases with increasing neutron energy. At the same time there is a high chance of absorption of neutrons by ^{238}U , which is not fissionable. it is therefore necessary to slow down, or moderate, these neutrons in order to increase their chances of initiating further fission process. the fissionable material is surrounded by a moderator, and the neutrons lose energy in collisions with the atoms of the moderator. When a neutron is scattered from a heavy nucleus like uranium, the loss of energy of neutron is negligibly small or nil. But when neutron collides with a light nuclei, the neutron can lose its substational amount of energy.

The main considerations for the choice of a moderator are (a) Mass comparable with that of neutrons and (b) low capture of neutrons. Graphite (Carbon) is there fore the best moderator which has a relatively small neutron absorption cross section. Enrico Fermi and his co-workers built the first nuclear reactor in 1942 at the University of Chicago. This reactor used carbon in the form of graphite blocks as moderator.

Ordinary water is frequently used as a moderator, because collisions with the protons are very effective in slowing the neutrons. However, neutrons have a relatively high probability of being absorbed by the water according to the reaction

$p + n \rightarrow {}^{2}_{1}H_1 + \gamma$. ${}^{2}_{1}H$ is called deuterium. In water hydrogen is replaced by deuterium, it is called heavy water. so heavy water is more useful as a moderator. it has virtually no neutron absorptions cross section. Many reactors use heavy water as moderator.

Control

To have continuous nuclear fission in reactors, the average number of neutrons in each fission react on that is available to produce next set of fission reactions must be exactly equal to one. If it is even slightly greater than one, the reaction rate will grow exponentially and out of control. Control of the reaction rate is accomplished by inserting the control rods made of cadmium. The cadmium has a very large cross section for absorbing neutrons and thus removing them from the fission process. Control rods are also used to shut down the reactor in case of an emergency. However, small fluctuations in the reaction rate occur much too rapidly so it is not possible to move control rods in and out to control the emitted neutrons instantly.

Fortunately, nature has provided us a solution to this difficulty. About 10% of the neutrons emitted in fission are delayed neutrons, produced not at the instant of fission but some what later, following radioactive decays of fission fragments. For

example Rubidium ^{93}Rb, which might be produced in the fission ^{235}U, beta decays with a half life of 6s to strontium (^{93}Sr), which is occasionally (in about 1% of decays) produced in a very excited state that is unstable to neutron emission. The neutron appears to emerge with the 6s half life of the beta decay. This short delay time is enough to allow the control rods to be adjusted to maintain a constant reaction rate.

The process is as follows. A uranium nucleus ^{235}U captures a neutron and splits into two heavy fragments and two prompt neutrons. One of the fragments emits a delayed neutron. The three neutrons are slowed by passage through the moderator. Two of the neutrons cause new fissions and the third is captured by ^{238}U, eventually to form fissionable plutonium ^{239}Pu, which can be recovered from the fuel by chemical means. See the figure below.
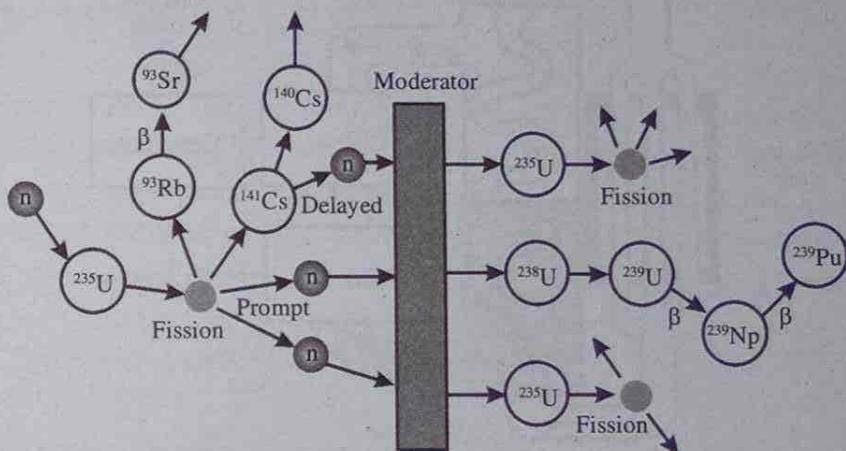
Figure 2.13: A typical sequence of processes in fission. A nucleus of ^{235}U absorbs a neutron and fissions; two prompt neutrons and one delayed neutron are emitted. Following moderation, two neutrons cause new fissions and the third is captured by ^{238}U, resulting finally in ^{239}Pu.

Fission reactors

Nuclear reactor is a source of intense heat. This heat is extracted to generate electric power. A fission reactor consists of mainly 3 parts.

1) Reactor core 2) Coolant system and 3) Shield

Reactor core

It contains fissionable material. Commonly used materials are ^{238}U enriched in ^{235}U and plutonium (^{239}Pu). The fissionable materials are made in the form rods about 1cm in diameter. To control the rate of fusion and also to shutdown the reactor

in the case of emergency, cadmium control rods are inserted at the middle of the reactor core. Water circulates inside the reactor core which act as the moderator.

Coolant system

A large amount of heat is produced in the reactor. This has to be extracted from the reactor core. This can be accomplished pressurised water reactor as shown in figure below. The heat is extracted in two step process. Water circulates through the core under great pressure. This will prevent turning boiled water into steam. This hot water then allowed to pass through another water system kept normal pressure. This will deliver steam to the turbine. Since steam never enters reactor core, it does not become radioactive, thus there is no radioactive material in the vicinity of the turbine.
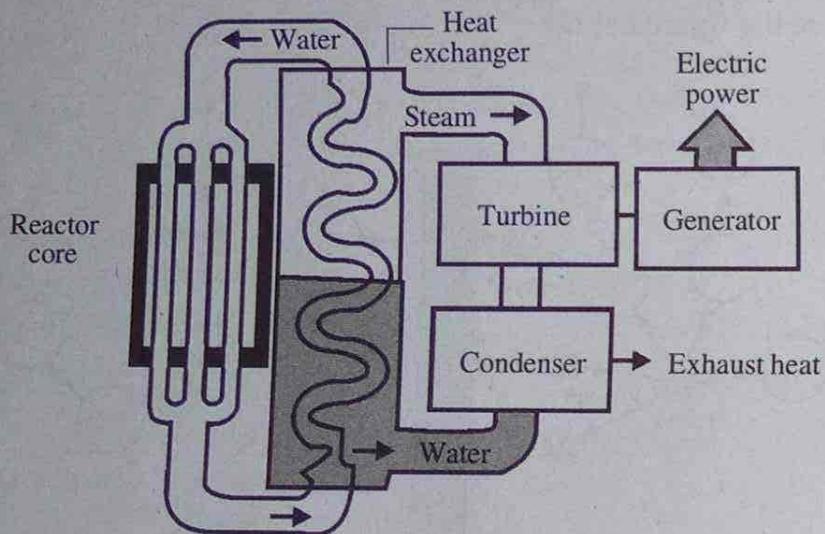

Figure 2.14: The components of a pressurized-water reactor

Shield

Various radiations emitted from the reactor which are injurious to the persons working near it. For their protection, the reactor core is surrounded by a radiation shield in the form of a thick concrete wall.

Almost all countries of the world use nuclear power plants. But there are some technological issues associated with it that are the subjects of active debate and investigation. Some of the radioactive isotopes among the fission fragments have very long half lives of the order of many years. The radio active waste from reactors must be stored in a manner that prevents leakage of radioactive material into the environment. Another botheration is about the resistance of reactors to natural disasters such as earth quakes, flooding or to acts of terrorism or sabotage.

There are two major disastrous and catastrophic incidents occurred in the history of nuclear reactors. In 1906, a graphite moderated power reactor of Chernobyl in the former U.S.S.R suffered a serious accident due to the disabling of the cooling system, which is designed to extract the intense heat generated in the reactor core. The resulting temperature rise ignited the graphite moderator and caused an explosion of the reactor containment vessel. It released radioactive fission products and exposing the inhabitants of the region to life threatening radiation loses. The water moderated reactors cannot suffer this kind of accident.

Another incident occurred in Japan. In 2011, the earth quake triggered tsunami that struck Japan's Fukushima reactor complex. The flooding of the reactor building caused the pumps supplying cooling water to fail. As a result, the reactor core over heated due to the radioactive decay of the fission products, and a partial meltdown of the fuel rods occurred. The release of radioactivity contaminated a wide region of the Japanese country side.

Fusion

Nuclear fusion is the phenomenon of two or more lighter nuclei to form to a single heavy nucleus. The mass of the product nucleus is less than the sum of the masses of the lighter nuclei fusing together. The difference in mass Δm results in the release of tremendous amount of energy according to Einstein's mass energy relation $\Delta E = \Delta mc^2$. In other words we can say that the energy released in this process is the excess binding energy of the heavy nucleus compared with the lighter nuclei. This process can release energy as long as the final nucleus is less massive than about $A = 60$.

For example consider the reaction

$$^2_1H + \,^2_1H \rightarrow \,^3_1H + \,^1_1H$$

The Q-value of the reaction can be estimated to be 4 MeV. (See example 10) so this reaction liberates about 1MeV per nucleon, roughly the same as the fission reaction. This reaction can occur when a beam of deuterons is accelerated onto a deuterium target. In order to take place the fusion, the incident and the target deuterons must be close enough such that nuclear force between them must overcome the electrostatic repulsion between them. The Coulombian repulsion energy can be estimated to be 0.5 MeV (see example 11). This means that a deuteron with 0.5 MeV of kinetic energy can initiate nuclear fusion and it can release an energy (4 MeV + 0.5 MeV) of 4.5 MeV.

Doing this reaction in an accelerator, in which the beam currents are in the micro ampere range, would produce only a small amount of energy. To obtain significant

amounts of energy it is necessary to work with much larger quantities of deuterium. For example the fusion energy from the deuterium in a litre of water (which contains 0.015% D_2O) would be equivalent to the chemical energy obtained from burning about 300 litres of gasoline.

The nuclear fusion can also take place under the conditions of high temperature and pressure. In this the deuterium gas is heated to a high temperature so that each atom of deuterium has about 0.25MeV of thermal kinetic energy. Then in the collision between two deuterium atoms, the total of 0.5 MeV of kinetic energy would be sufficient to overcome the Coulomb repulsion. Hence this is called thermo nuclear fusion.

The temperature corresponding to 0.25 MeV energy is

$$\frac{1}{2}kT = 0.25 \text{ MeV}$$

$$\frac{1}{2}kT = 0.25 \times 1.6 \times 10^{-13}$$

$$\therefore \quad T = \frac{2 \times 0.25 \times 1.6 \times 10^{-13}}{1.38 \times 10^{-23}} = 0.58 \times 10^{9} \text{K}.$$

It is very difficult to produce temperature of the order of 10^{9}K. However this temperature would exist in the interior of stars where energy is produced due to nuclear fusion. Scientists and engineers are in the hot pursuit of producing nuclear fusion energy in the laboratories for generating electric power small scale production of fusion energy for a small scale of time is attained in laboratories. We are waiting for a sustained and controllable fusion reactor that can deliver constant output. It is not yet a reality. A great deal of effort is currently underway to resolve various difficulties in the development of a successful device. Nevertheless controlled fusion is regarded as the ultimate energy source because of the abundant availability of its main fuel: water.

Example 10

Calculate the Q – value of the reaction: $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{1}H + ^{1}_{1}H$

$m(^{2}H) = 2.014102 \text{ u}, \quad m(^{3}H) = 3.016029 \text{ u}$ and $m(^{1}H) = 1.007825\text{u}$

Solution

$$Q = [2m(^2H) - m(^3H) - m(^1H)]c^2$$

$$Q = [2 \times 2.014102\,u - 3.016029\,u - 1.007825\,u]c^2$$

$$Q = 0.0435uc^2$$

or $\qquad Q = 0.0435 \times 931.5\text{ MeV} = 4.04\text{ MeV}$

Example 11

Calculate the Coulomb energy between two deuterons (^2_1H), when they just touch each other.

Solution

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 R}$$

$$U = \frac{2e2e}{4\pi\varepsilon_0 R}$$

Radius of deutron $r = r_0 A^{1/3} = 1.2(2)^{1/3}$ fm

$$r = 1.512 \text{ fm}$$

$\therefore \qquad R = 2r = 2 \times 1.512\text{fm} = 3.024 \times 10^{-15}\,m$
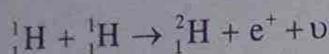
$\therefore \qquad U = \dfrac{9 \times 10^9 \times (2 \times 1.6 \times 10^{-19})^2}{3.024 \times 10^{-15}}\,J$

$$U = \frac{9 \times 10^9 \times (2 \times 1.6 \times 10^{-19})^2}{3.024 \times 10^{-15} \times 1.6 \times 10^{-13}}\,\text{MeV}$$

$$U = \frac{9 \times 4 \times 1.6 \times 10^{-1}}{3.024} = 0.629 \text{ MeV}$$
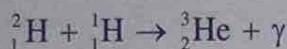
Fusion processes in stars

Stars are composed of ordinary hydrogen rather than deuterium. Nuclear fusion to take place, it is necessary to produce deuterium. This is done according to the reaction.

$$^1_1H + ^1_1H \rightarrow ^2_1H + e^+ + \upsilon$$
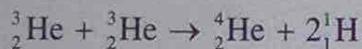
i.e., proton combines with proton giving one deuterium, a positron and a neutrino. This is analogues to β^+ decay in which one proton is converted to a neutron, a positron and a neutrino.
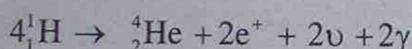
Next reaction that occurs is the deuterium combines with another proton resulting in ^3_2He and a gamma ray according to:

$$^2_1H + ^1_1H \rightarrow ^3_2He + \gamma$$
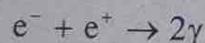
For the next reaction to occur we need one more ^3_2He. For this first two reaction must occur twice. Finally ^3_2He combines with ^3_2He forming ^4_2He and two protons. Accordingly we have

$$^3_2He + ^3_2He \rightarrow ^4_2He + 2^1_1H$$

In the overall processes, four protons are involved. So the net process can be written as

$$4^1_1H \rightarrow ^4_2He + 2e^+ + 2\upsilon + 2\gamma$$

To calculate the Q-value, all nuclear masses must be converted into atomic masses. For this add $4m_e$ on both sides. On the L.H.S four protons added with 4 electrons giving four neutral hydrogen atom. On the R.H.S two electrons added with ^4_3He nuclei giving a neutral helium atom. The remaining two electrons combines with two positrons giving additional gamma rays.

$$e^- + e^+ \rightarrow 2\gamma$$

This will give additional energy from the reaction.

According to definition, the Q-value is

$$Q = (m_i - m_f)c^2 = [4m(^1_1H) - m(^4_2He)]c^2$$

Substituting the values of atomic masses of hydrogen and helium, we get

$$Q = [4 \times 1.007825\,u - 4.002603\,u]c^2$$

$$Q = 0.028697uc^2$$

Using $1u = 931.5 \dfrac{MeV}{c^2}$

$$Q = 0.028697 \times 931.5 \, \text{Mev}$$

$$Q = 26.73 \, \text{MeV}$$

This shows that each fusion liberates about 26.73 MeV of energy.

Now we will see at what rate these reactions must occur in the Sun? Earth receives 1400 joules of energy in every second from the sun by an area 1m². This is called the solar constant. So the total energy received by overall area $4\pi r^2$ is $4\pi r^2 \times 1400$. This comes about 4×10^{26} joules (where $r = 1.5 \times 10^{11}$ m, the distance between the sun and the earth). In other words the power output of the sun is 4×10^{26} W. Converting this into MeV, we get $\dfrac{4 \times 10^{26}}{1.6 \times 1^{-13}} \dfrac{\text{MeV}}{\text{s}}$. This is about 2.5×10^{39} MeV / s. Each fusion reaction liberates about 26 MeV and thus there must be $\dfrac{2.5 \times 10^{39}}{26} \simeq 10^{38}$ fusion reactions per second. This will consume about 4×10^{38} protons per second. The sun's mass is about 2×10^{30} kg which corresponds to about 10^{57} protons. This is enough to burn for the next few billion years.
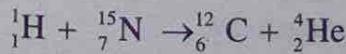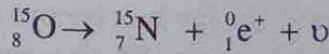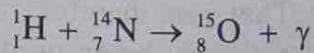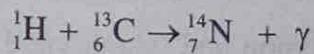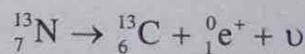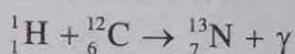
The sequence of reactions described above is called proton-proton cycle. However it is probably not the primary source of fusion energy in many stars. This is because the first reaction which is similar to β^+ decay takes place only on a very long time scale and is therefore unlikely to occur. The carbon cycle is believed to be the more likely sequence of reactions.

Carbon cycle reactions

The cycle proceeds as follows:

$$^{1}_{1}\text{H} + ^{12}_{6}\text{C} \rightarrow ^{13}_{7}\text{N} + \gamma$$

$$^{13}_{7}\text{N} \rightarrow ^{13}_{6}\text{C} + ^{0}_{1}\text{e}^{+} + \upsilon$$

$$^{1}_{1}\text{H} + ^{13}_{6}\text{C} \rightarrow ^{14}_{7}\text{N} + \gamma$$

$$^{1}_{1}\text{H} + ^{14}_{7}\text{N} \rightarrow ^{15}_{8}\text{O} + \gamma$$

$$^{15}_{8}\text{O} \rightarrow ^{15}_{7}\text{N} + ^{0}_{1}\text{e}^{+} + \upsilon$$

$$^{1}_{1}\text{H} + ^{15}_{7}\text{N} \rightarrow ^{12}_{6}\text{C} + ^{4}_{2}\text{He}$$

The overall result is the combination of four hydrogen nuclei to form a helium nucleus plus two positrons in the same way as in the proton-proton cycle. This reaction neither consumes nor produce any $_6^{12}C$. $_6^{12}C$ plays the role as a catalyst. The Q value is the same as before (26.73MeV). (See example 12)

The Coulomb repulsion between $_1^1H$ and $_6^{12}C$ is larger than the Coulomb repulsion between $_1^1H$ and $_1^1H$, so more thermal energy and a correspondingly higher temperature are needed for the carbon cycle. The carbon cycle is important at a temperature about $20 \times 10^6\,K$, while the Sun's interior temperature is only about $15 \times 10^6\,K$.

The symbolic diagram of proton-proton cycle and carbon cycle are given below.

(a) The proton-proton cycle   (b) The carbon cycle
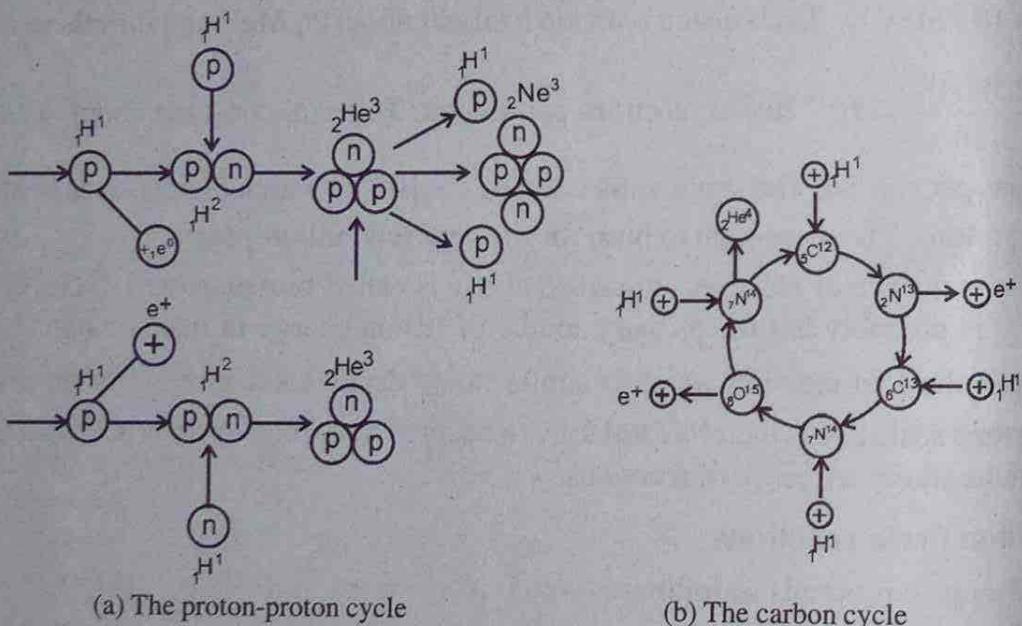
Figure 2.15

Fusion reactors

We know that both the reactors of nuclear fission and fusion yield much more energy than other conventional nuclear reactions. They can, therefore, be put to use to generate energy. But fission based reactors are already being used to generate electricity, fusion based controlled reactors have so far not become a practical possibility. They are, however, reactors of the future because of their many advantages over the fission reactions. Some of these advantages are.

1. The basic fuel for nuclear fusion - hydrogen and deuterium is available in unlimited quantities in sea water. The most important thing is that deuterium is present in sea water is cheap to extract. The concentration of deuterium in sea water-is only $33g/m^3$ and this add upto a total of about 10^{15} tonnes of deuterium in the worlds ocean. The deuterium in a gallon of sea water can yield as much energy through fusion as 600 gallons of gasoline can through combustian.

2. The energy yield per unit mass of the fuel is much more in fusion than in fission.

3. There are no radioactive pollutants arising as a by product of the fusion reactions. On the other hand, fission results in some radio active isotopes that are quite long lived and, therefore, almost a permanent source of radioactive pollution.

Because of these advantages, scientists are keenly exploring possibility of a fusion reactor. It is clear that fusion holds the promise of meeting the energy requirements challenge of the future. Hence fusion reactors are indeed the reactors of future.

There are mainly two types of nuclear fusion reactions. They are deutron-deutron (D-D) reaction and deutron - triton (D-T) reaction.

D-D reactions
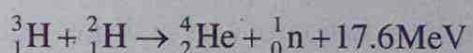
In this two deuterons fuse to form a triton and a proton and energy. $_1^2H + _1^2H \rightarrow _1^3H + _1^1H + 4MeV$ or two deutrons combine to form a $_2^3He$ nucleus and a neutron and energy.
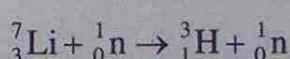
$$_1^2H + _1^2H \rightarrow _2^3He + _0^1n + 3.3MeV$$. Both D-D reactions have about equal probabilities.

D-T reaction

In this a deutron and a triton fuse to form a helium nucleus and a neutron with the release of energy.

$$_1^3H + _1^2H \rightarrow _2^4He + _0^1n + 17.6MeV$$

D-T reaction releases much more energy than D-D reaction. But the availability of tritium is difficult and sea water contains too little tritium to be extracted economically. However it can be produced by the neutron bombardment of the two isotopes of natural lithium.

$$_3^6Li + _0^1n \rightarrow _1^3H + _2^4He$$

$$_3^7Li + _0^1n \rightarrow _1^3H + _0^1n$$

Basic requirements of fusion reactor

1.   During fusion processes very strong coulomb repulsion exist between the two fusing nuclei. Hence very large kinetic energies, of the order of 1MeV, are needed so that the nuclei get close enough for the attractive nuclear forces to become effective and cause fusion. These large kinetic energies can be obtained in an accelerator.

2.   To obtain energy from fusion, the particles must be heated to a temperature great enough for the fusion reaction to occur as the result of random thermal collisions. Because of thermal energy some particles can overcome the coulomb barrier. A temperature T corresponding to $kT = 10 keV$ is required for a reasonable number of fusion reactions to occur. The temperature corresponding $kT = 10 eV$ is of the order of $10^8 K$. Such temperatures occur in the interiors of stars. At these temperatures, a gas consists of positive ions and negative electrons called plasma. The main problem lies in confining the plasma long enough for the reactions to take place. The enormous gravitational field of the sun confines plasma in the interior of the sun.

3.   The energy required to heat a plasma is proportional to the density of its ions n. The fusion rate is proportional to square of density n^2. The output energy is proportional to $n^2\tau$, where τ is the confinement time. If the output energy is to exceed the input energy, we must have

$$C_1 n^2 \tau > C_2 n$$

where C_1 and C_2 are constants. Lawson derived the following relation between density and confinement time, known as Lawson's criterion

$$n\tau > 10^{20} \text{ sm}^{-3}$$

The product $n\tau$ is called the confinement quality parameter. When the above criterion is met the energy released by a fusion reactor will just equal to the energy input, hence called break even condition. For this to happen the thermal energy of the ions is great enough. In the case of D-T plasma this is about 10keV. For a feasible reactor much more energy must be released.

Confinement methods

We have found that for nuclear fusion to take place plasma must be confined.

Magnetic Confinement

A magnetic field can confine a plasma because the charged particles spiral around

the magnetic field lines. Figure below shows a toroidal magnetic confinement. There are two contributions to the magnetic field: one is along the toroid axis another is around the axis. The combination of these two gives a helical field along the toroid axis, and the charged particles are confined as they spiral about the field lines. This type of device is called a tokamak (from the Russian acronym for toroidal magnetic chamber. A current passed through the plasma serves both ot heat the plasma and to create one of the magnetic components.

Field coils   Magnetic
                           field lines
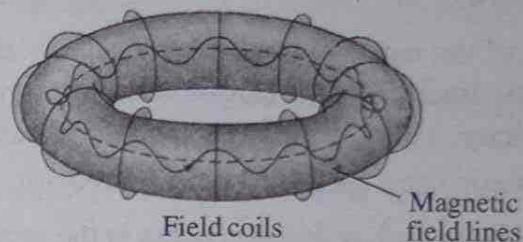
Figure 2.16: The toroidal geometry of plasma confinement. The ionized atoms circulate around the ring, trapped by the magnetic field lines. The coils produce a magnetic field along the axis of the toroid (dashed line). Another field component is produced by a current along the axis in the plasma. The two components of the field produce the helical field lines shown.
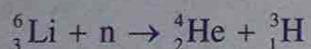
The Tokamak Fusion Test Reactor (TFTR) at Princeton University, which operated from 1982 to 1997 achieved an ion temperature of $5.1 \times 10^8 \,\mathrm{K}$ and a fusion power level of 10.7 MW. This came very close to reaching Lawson's criterion with a plasma density of $n = 10^{20}$ particles per m³ and a confinement time of $\tau = 0.2\mathrm{s}$

Inertial confinement

In this confinement process a small pellet of D.T fuel is compressed to high densities for very short confinement times by striking the pellet from many directions with intense laser beams. This will first vapourise the pellet and convert it to a plasma, and then heat and compress it to the point at which fusion can occur. The laser pulses last only about 1 ns. Thus according to Lawson's criteria the density must exceed 10^{29} particles per m³. ($n\tau \geq 10^{20}$ and $\tau = 10^{-9}\mathrm{s}$). However, because of inefficiencies of the lasers and other losses a self sustaining laser fusion reactor must exceed this minimum by about 2-3 orders of magnitude. In 2010 the first inertial confined system was operated at the Lawrence Livermore National Laboratory (LLNL). It is designed so that a 2mm diameter pellet of D.T is struck simultaneously by 192 laser beams that deliver an energy of 1MJ in a pulse lasting a few nanosecond, which is expected to compress the pelllet to a central density that is 100 times

that of lead. In June 13, 2018 an experimental campaign conducted at LLNL, achieved a total fusion energy output of 54kJ.

The main difficulty in the D-T fusion reaction is the recovery of the energy from the neutrons and its conversion into electrical power. One possibility for fusion reactor design is that the reaction area is surrounded by lithium, which captures neutrons by the reaction

$$_3^6 Li + n \rightarrow {}_2^4 He + {}_1^3 H$$
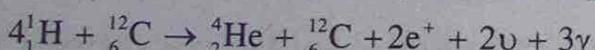
The kinetic energies of the reaction products are rapidly dissipated as heat, and the thermal energy of the liquid lithium can be used to convert water to steam in order to generate electricity. This reaction has the added advantage of producing tritium ($^3 H$) which is needed as a fuel for the fusion reactor.

Another difficulty with the D-T reaction process is the production of large number of neutrons. Although fusion reactors will not produce the radio active wastes that fission reactors do, the neutrons make the immediate area surrounding the reactor radioactive. This results in the damage of reactor vessel. Here once again the lithium is helpful, because a 1m thickness of lithium should be sufficient to stop essentially all of the neutrons.

Fusion energy is the subject of vigorous research all over the world. All technological problems are being attacked with a variety of methods. Researchers are hopeful that solutions can be found out soon, so that fusion can help to supply our electrical power requirements.

Example 12

Calculate the Q-value of the carbon cycle reaction process:

$$4_1^1 H + {}_6^{12} C \rightarrow {}_2^4 He + {}_6^{12} C + 2e^+ + 2\upsilon + 3\gamma$$

$$m(_1^1 H) = 1.007825 \text{ u}, \quad m(^4 He) = 4.002603 \text{ u}$$

Solution

$$Q = [4m(_1^1 H) - m(_2^4 He)]c^2$$

$$Q = (4 \times 1.007825u - 4.002603u)c^2$$

$$Q = 0.028697uc^2$$

$$Q = 0.028697 \times 931.5 \text{MeV}$$

$$Q = 26.73 \text{ MeV.}$$

This is same as the Q-value of proton-proton cycle.

Example 13
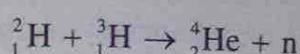
Show that the D-T fusion reaction releases 17.6 MeV of energy.

$$m(^2H) = 2.014102u, \quad m(^3H) = 3.016049u,$$

$$m(^4He) = 4.002603u \text{ and } m_n = 1.008665u$$

Solution

D-T Fusion reaction is

$$^2_1H + {}^3_1H \rightarrow {}^4_2He + n$$

The Q-value is:

$$Q = [m(^2H) + m(^3H) - m(^4He) - m_n]c^2$$

$$Q = [2.014102u + 3.016049u - 4.002603u - 1.008665u]c^2$$

$$Q = 0.018883uc^2$$

$$Q = 0.018883 \times 931.5 MeV$$

$$Q = 17.5895 \text{ MeV}$$

$$Q = 17.6 \text{ MeV}$$

Example 14

If a tokamak fusion reactor were able to achieve a confinement time of 0.60s, what minimum particle density is required.

Solution

a) $\tau = 0.60$ s

We have

$$n\tau \geq 10^{20}$$

$$\therefore \qquad n \geq \frac{10^{20}}{0.6} = 1.67 \times 10^{20} \text{ m}^{-3}$$

$$n = 1.67 \times 10^{21} \text{ m}^{-3}$$

Applications of nuclear physics

So far we found only two main applications one is the estimation of the age of the rocks and woods, the other one is the production of electric power. Besides these there are somany applications of nuclear physics. Here we discuss only three among them.

Neutron activation analysis

Neutron Activation Analysis (NAA) is a method to determine the concentrations of elements in a given substance.

The technique involves irradiating the material to be analysed with a neutron beam. The irradiation will result in the absorption of neutrons by various nuclei of the elements in the material yielding many radio isotopes. The energies and intensities of the gamma-ray emissions of the radio isotopes produced are then measured with a sensitive semiconductor detector and the elements in the sample as well as their concentrations are determined by a correlating the gamma ray energy lines and their intensities with standards irradiated simultaneously with the unknown material.

For example ^{59}Co is placed in a flux of neutrons, neutron absorption results in the production of radioactive isotope ^{60}Co. ^{60}Co undergoes beta decay with a half life of 5.27 years. Following the beta decay, nickel ^{60}Ni emits two gamma -rays of energies 1.17 MeV and 1.33 MeV and of equal intensity. If an unknown material is placed in a flux of neutrons and observe the spectrum. If the spectrum contains 1.17 MeV and 1.33 MeV, it is certain the unknown material contains cobalt.

NAA has high level of sensitivity for the trace elements ($< 0.01\%$) and accuracy it is commonly employed in many fields of study including the biological sciences, agriculture, industry food and nutrition. It is also commonly used in forensic science because of characteristic signatures that the gamma spectra will display after neutron irradium of samples.

There are two main drawbacks to the use of NAA. Even though the technique is essentially non destructive, the irradiated sample will remain radioactive for many years after initial analysis. This requires careful handling and disposal of this radioactive material.

An example of a neutron activation analysis study of a sample of hair is shown in figure.
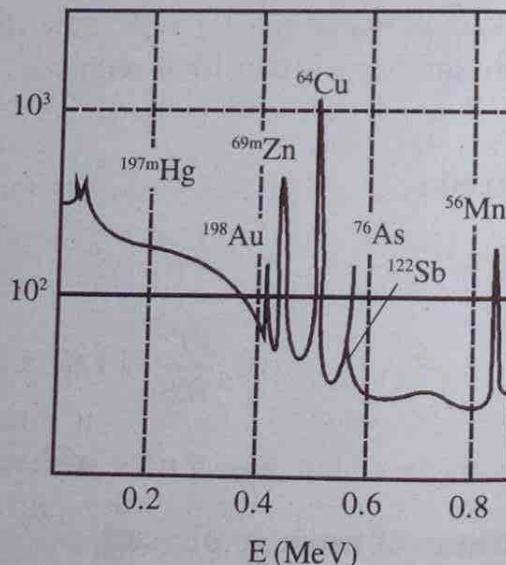

Figure 2.17: Gamma-ray spectrum following neutron activation of a sample of human hair. The sample shows traces of mercury, gold, zinc, copper, arsenic, antimony, and manganese.

Medical radiation physics

One of the most important applications of nuclear physics is in the medical field for diagnostic and therapeutic purposes. The use of X-rays for producing images for medical diagnosis is known to everybody. Though X-rays show distinct and detailed images of bones, they are unable to take photographs of soft issues inside the human body. To take the photographs of soft tissues radioactive isotopes are introduced into the human body in chemical forms that have an affinity for certain organs, such as bone or thyroid gland. A sensitive detector (gamma ray camera) can observe the radiations from the isotopes that are concentrated in the organ and can produce an image that shows how the activity is distributed in the patient. These detectors are capable of determining where each gamma -ray photon originates in the patient.

Another technique used in medical diagnosis is called positron emission tomography (P E T). This reveals a wealth of information. Positron emission tomography is an imaging technique that uses radioactive substances to visualise and identifying changes at the cellular level.

PET uses small amounts of radioactive materials called radio tracers, a special camera and a computer to evaluate organ and tissue functions. By identifying changes at the cellular level, PET may detect the early onset of disease before other imaging tests can.

Functioning of PET

Radio tracers (Positron emitting isotope) are either swallowed, inhale or injected into the vein depending on what part of the body is being examined. Certain organs and tissues then absorb the tracer. The tracer will collect in areas of higher chemical activity which is helpful. Because certain tissues of the body and certain diseases have a higher level of activity. These areas of disease will show up as bright spot in the PET scan.

Usually radio tracers used are ^{15}O ($T_{1/2}$ = 2 minutes) ^{13}N ($T_{1/2}$ = 10 minutes) ^{11}C ($T_{1/2}$ = 20 minutes) and ^{18}F ($T_{1/2}$ =110 minutes). Since these isotopes have short half-lives this must be produced at the site of the diagnostic facility by a cyclotron. When a positron emitter decays, the positron quickly annihilates with an electron and produces two 511 keV gamma -rays that travel in opposite directions. By surrounding the patient with a ring of detectors, it is possible to determine exactly where the decay occurred and from a large number of such events the physician can produce an image that reconstructs the distribution of radioisotope in the patient. One advantage of PET is that it can produce a dynamic image-changes in the patient during the measuring time can be observed.

In radiation therapy uses the effect of radiations in destroying unwanted tissue in the body such as a cancerous growth or an overactive thyroid gland. When radiation is passed through matter it ionises the atoms. The ionised atoms then participate in chemical reactions that lead to their incorporations into molecules and subsequent alteration of their biological function, possibly the destruction of a cell or the modification of its genetic material. For example, an overactive thyroid gland is often treated by giving the patient radioactive iodine (^{131}I), which collects in the thyroid. The beta emissions from the isotope damage the thyroid cells and ultimately lead to their destruction. Certain cancers are treated by implanting needles or wires containing radium or other radioactive substances. The decays of these radioisotopes cause localised damage to the cancerous cells.

Other cancers can be treated using beams of particles that cause nuclear reactions within the body at the location of the tumor. Pions and neutrons are used for this purpose. The absorption of a pion or a neutron by a nucleus causes a nuclear reaction, and the subsequent emission of particles or the reaction products again causes local damage that is concentrated at the site of the tumor, inflicting maximum damage to the tumor and minimum damage to the surrounding healthy tissue.

Alpha scattering application

Radioactive sources emitting alpha particles have been used in a variety of ways. Alpha particles emit at a fixed rate in any location. This property is made use of in most of the applications.

1) Alpha particle emission can be used to produce electric power. The energy of the alpha particle absorbed can be converted into electric power through thermo electric power. The power is of the order of one watt per gram material (see example 15). These powers are sufficient to operate many devices such as cardiac pace makers, voyager space craft which photographed Jupiter, Saturn and Uranus etc.

2) Alpha particles are used in ionisation type smoke detectors. Alpha particles from the decay of Americium (^{241}Am) are scattered by the ionised atoms that comes from smoke. When the smoke detector senses a decrease in the rate at which alphas are counted, the alarm is triggered.

3) In Rutherford scattering alpha particle emission enables us to calculate the change in kinetic energy of the scattered alpha particles that scatters through 180° using
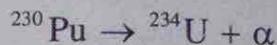
$$\Delta K = K \frac{4m/M}{\left(1 + \dfrac{m}{M}\right)^2}$$

This can be used for material analysis (See example 16). The survey or space craft that landed on the Moon and the Viking landed on Mars carried Rutherford back scattering experiments to analyse the chemical composition of the surfaces of those planets.

Example 15

A radioactive source is to be used to produce electric power from the alpha decay of plutonium ^{238}Pu($T_{1/2} = 88y$). What is the Q-value for the decay? (b) Assuming 100% conversion efficiency, how much power could be obtained from the decay of 1.0g of ^{238}Pu. $m(Pu) = 238.049560$ u . $m(U) = 234.040952$ and $m(He) = 4.002603u$

Solution

$$^{230}Pu \rightarrow {}^{234}U + \alpha$$

a) $Q = [m(Pu) - m(U) - m(^4He)]c^2$

     $Q = [238.049560u - 234.040952u - 4.002603u]c^2$

     $Q = 0.06005u\,c^2$

     $Q = 0.06005 \times 931.5$ MeV

     $Q = 5.593$ MeV

b) $1.0g = \dfrac{1}{238}$mole $= \dfrac{1}{238} \times 6.22 \times 10^{23}$ atoms

         $= 2.53 \times 10^{21}$ atoms.

    $\therefore$ Activity, $a = \lambda N = \dfrac{0.693}{T_{1/2}} N$

     $a = \dfrac{0.693 \times 2.53 \times 10^{21}}{88 \times 365 \times 24 \times 60 \times 60}$

     $a = 6.32 \times 10^{11}$ s^{-1}

    $\therefore$ Power, $P = aQ = = 6.32 \times 10^{11} \times 5.593 \times 1.6 \times 10^{-13} \dfrac{J}{s}$

         $P = 0.566W$

Example 16

An alpha particle of mass m makes an elastic head on collision with an atom of mass M at rest. Show that the loss in kinetic energy

$$\Delta K = K \left[\frac{4m/M}{\left(1+\dfrac{m}{M}\right)^2} \right]$$

Solution
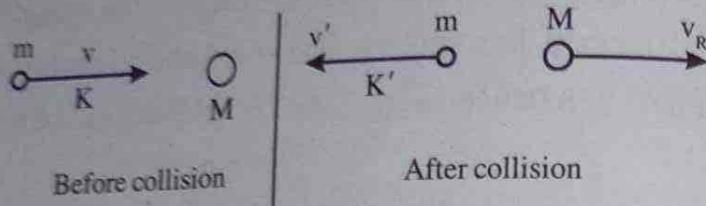
Before collision | After collision

Figure 2.18

Momentum conservation gives

$$mv = -mv' + Mv_R \qquad \dots (1)$$

Energy conservation gives

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}Mv_R^2 \qquad \dots (2)$$

or

$$K = K' + \frac{1}{2}Mv_R^2$$

From eq. (1), we get

$$m(v+v') = mv_R \qquad \dots (3)$$

From eq. 2, we have

$$m(v^2 - v'^2) = Mv_R^2 \qquad \dots (4)$$

$\dfrac{Eq\,4}{Eq\,3}$ gives

$$v - v' = v_R \qquad \dots (5)$$

solving eqs 3 and 5 we get

$$v' = v\,\frac{(1-m/M)}{(1+m/M)}$$

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2$$

$$\Delta K = \frac{1}{2}mv^2\left(1-\frac{v'^2}{v^2}\right)$$

$$\Delta K = \frac{1}{2}mv^2\left[\frac{\left(1-\frac{m}{M}\right)^2}{\left(1+\frac{m}{M}\right)^2}\right] = \frac{4m/M}{\left(1+\frac{m}{M}\right)^2}$$

Synthetic elements

Synthetic elements are chemical elements that do not occur naturally on earth. There are totally 24 synthetic elements that have been created. They are created in nuclear reactors, particle accelerators or in the explosion of atom bombs. Since they are artificially (or man made) created they are called as synthetic elements.

The mechanism for the creation of synthetic elements is to force protons or neutrons onto the nucleus of an element with atomic number less than 95. All synthetic elements are unstable but they decay at widely varying rate. Their half-lives range from 15.6 million years to a few hundred micro seconds. The names of the synthetic elements are given, in the increasing order of atomic number, below in the table 2.1

Table 2.1: Synthetic elements

No.	Atomic number	Name of the element	No	Atomic number	Name of the element
1	43	Technetium	13	106	Seaborgium
2	61	Promethium	14	107	Bohrium
3	85	Astatine	15	108	Hassium
4	87	Francium	16	109	Meitnerium
5	95	Americium	17	110	Darmstadtium
6	99	Einsteinium	18	111	Roentgenium
7	100	Fermium	19	112	Copernicium
8	101	Mendelevium	20	113	Nihonium
9	102	Nobelium	21	114	Flerovium
10	103	Lawrencium	22	115	Moscovium
11	104	Rutherfordium	23	116	Livermorium
12	105	Dubnium	24	118	Oganesson

The known atoms beyond uranium ($Z = 92$) are all radioactive, with half-lives compared with the age of the earth. Therefore they are not present on earth, but they can be produced in the laboratory. The chemical elements with atomic number greater than 92 are called transuranium elements. The atomic number starting at 93 and ending at 118 are transuranium elements. Sometimes chemical elements with atomic number greater than 103 are called transactinides.

The process of neutron capture followed by beta decay is used to produce elements with $Z = 93$ to $Z = 100$. Beyond this accelerators, reactors, atom bomb (mini) explosions are required. All elements upto $Z = 118$ have been observed and named.

Elements with atomic number greater 99 have no use other than scientific research. Since they have extremely short half-lives and thus never been produced in large quantities.

IMPORTANT FORMULAE

1. Reaction probability:

   a) $R = \dfrac{\sigma N I_0}{S}$ b) $Q = \sigma \phi \dfrac{m N_A}{M}$

2. Production of activity in reaction:
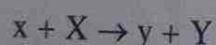
   $$a(t) = \lambda N = R(1 - e^{-\lambda t})$$

   Where $\lambda = \dfrac{0.693}{T_{1/2}}$

3. Fraction of the maximum possible activity:

   $$f = \dfrac{a(t)}{R} = 1 - e^{-\lambda t}$$

4. Reaction Q-value:

   $$x + X \rightarrow y + Y$$
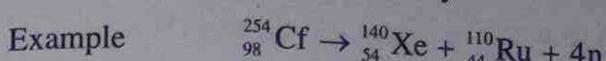
   $$Q = (m_i - m_f)c^2$$

   $$Q = \left[(m(x) + m(X) - m(y) - m(Y)\right]c^2$$
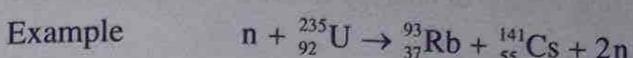
   $$Q = K_y + K_Y - K_x$$

5. Expression for threshold kinetic energy $K_{Th} = -Q\left[1 + \dfrac{m(x)}{m(X)}\right]$

6. Spontaneous nuclear fission decay

   Example $^{254}_{98}Cf \rightarrow \, ^{140}_{54}Xe + \, ^{110}_{44}Ru + 4n$

7. Induced nuclear fission decay

   Example $n + \, ^{235}_{92}U \rightarrow \, ^{93}_{37}Rb + \, ^{141}_{55}Cs + 2n$
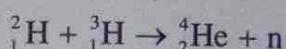
8. Expression for critical energy

$$E_{cri} = 0.89 \, A^{2/3} - 0.02 \frac{Z(Z-1)}{A^{1/3}} \, MeV$$

9. D-T fusion reaction

$$^{2}_{1}H + \, ^{3}_{1}H \rightarrow \, ^{4}_{2}He + n$$

10. Lawsons criterion

$$n\tau \geq 10^{20} \, m^{-3}s$$

11. Change in kinetic energy in alpha back scattering:

$$\Delta K = K \left[\frac{4\dfrac{m}{M}}{\left(1 + \dfrac{m}{M}\right)^2} \right]$$

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in about* **two** *or* **three** *sentences*)

Short answer type questions

1. How the study of nuclear reaction is more important than the study of nuclear decays?

2. What is a nuclear reaction?

3. A nuclear reaction doesn't involve beta decay. Justify?

4. What are the observations that we make in a nuclear reaction experiment?

5. How nuclear reaction experiment is conducted in laboratories?

6. What is reaction cross section? What is its unit?

7. How do we produce radioisotopes in nuclear reactions?

8. Write down an expression for the activity of a radioisotope produced in nuclear reaction and explain the symbols?

9. Write down an expression for the Q-value of a nuclear reaction and explain the symbols?

10. What is meant by threshold kinetic of a projectile involved in nuclear reaction?

11. Write down an expression for threshold kinetic energy and explain the symbols?
12. What is nuclear reaction? Give an example?
13. What is meant by critical energy of nuclear fission?
14. Write down an expression for critical energy of nuclear fission and explain the symbols used?
15. What is induced nuclear fission?
16. Ordinary uranium is not fissionable. Then what are main things required to make it fissionable?
17. What is the use of cadmium rods in nuclear reactors?
18. What is the function of moderators in nuclear reactions?
19. Write down two names of moderators?
20. Why ^{238}U is not suitable for nuclear fission?
21. Which are the main parts of a nuclear reactor?
22. What is the function of shield in a nuclear reactor?
23. What was the cause of Chernobyl accident of nuclear reactor?
24. What is nuclear fusion? Where does the energy come from?
25. What are the conditions under which nuclear fusion occurs?
26. "Controlled nuclear fusion is the ultimate energy source of mankind", Justify?
27. What is tokamak?
28. What is Lawson's criteria?
29. What is neutron activation analysis?
30. Give any three uses of neutron activation analysis?
31. What does PET stand for?
32. What are radio tracers? Give two examples?
33. Write down two uses of medical radiation physics?
34. How does alpha particle emission use to produce electric power?
35. What are synthetic elements?
36. Give three names of synthetic element?
37. What are the properties of synthetic elements?
38. What are transuranic elements?
39. What are transactinoids?
40. How does synthetic element whose mass number 99 is prepared in laboratory?

Section B
(*Answer questions in a* **paragraph** *of about half a page to one page*)

Paragraph / Problem type questions

1. How will you measure the outgoing particle energy in a nuclear reaction experiment?
2. How will you evaluate the reaction probability in a nuclear reaction experiment?
3. Derive an expression for reaction cross section?
4. Derive an expression for activity of a radioactive isotope in the process of its production?
5. What are exothermic and endothermic reaction?
6. Explain what is meant by spontaneous nuclear fission?
7. Explain what is meant by artificial nuclear fission?
8. Explain the mechanism behind nuclear fission?
9. The critical energy for the ^{239}U compound nucleus is 7.3 MeV. Justify?
10. What is meant by enrichment of natural uranium?
11. What is meant by moderation in reference to nuclear fission?
12. How do we control nuclear fission processes?
13. What are the main problems associated with nuclear reactors?
14. What are the essential components of reactor core?
15. What is the function of a coolant system in a nuclear reactor?
16. Explain the fusion process in stars?
17. Calculate the number of fusion reactions that occurs in the sun in one second?
18. Explain the proton-proton cycle reaction with an example?
19. Explain the carbon-nitrogen cycle reaction with an example?
20. Explain D-D reaction with an example?
21. Explain D-T reaction with an example?
22. Write down three basic requirements of fusion reactor?
23. What are the advantages of nuclear fusion reactors over nuclear fission reactors?
24. Explain the magnetic confinement of plasma?
25. Explain the inertial confinement of plasma?
26. Write a short note on the future energy source of reactors?
27. Explain the technique of neutron activation analysis?
28. Explain the technique of taking photograph of soft tissues inside a human body?
29. How does positron emission tomography function?
30. How does radiation effect on cancerous tumor?
31. How does an overactive thyroid is treated with radiation?
32. How alpha particle emission is used in smoke detectors?

33. Write brief note on synthetic elements?

34. Fill up the missing particle in the reactions given below.
    a) $^4He + {}^{14}N \rightarrow {}^{17}O + -...$
    c) $^{27}Al + {}^4He \rightarrow n + ...$
    b) $^9Be + {}^4He \rightarrow {}^{12}C + -...$
    d) $^{12}C + ... \rightarrow {}^{13}N + n$

    [a) ^1_1H b) ^1_0n c) $^{30}_{15}P$ d) ^2_1H]

35. In a certain nuclear reaction, outgoing protons are observed with energies 16.2 MeV, 14.8 MeV, 11.6 MeV, 8.9 MeV and 6.7 MeV. No energies higher than 16.2 MeV are observed. Construct a level scheme of the product nuclear?

36. For a certain incident proton energy the reaction $P + {}^{56}Fe \rightarrow n + {}^{56}Co$ has a cross section of 0.40b. If we bombard a target in the form of a $1.00cm^2$, $1.00\,\mu m$ thick iron foil with a beam of protons equivalent to a current of $3.00\,\mu A$, and if the beam is spread uniformly over the entire surface of the target. At what rate the neutrons are produced. Density of iron 7.90 gcm^{-3} $(6.37 \times 10^7$ particles per second$)$

37. In order to determine the cross section for neutron capture, you are irradiating a thin gold foil in the form of a circular disc of diameter 3.00mm and thickness $1.81\,\mu m$; with neutrons to produce the reaction $n + {}^{197}Au \rightarrow {}^{195}Au + \gamma$. By observing the outgoing gamma-ray photons in a detector, you determined that the gold decays at a rate of 5.37×10^6 per second. From an independent measurement, you have determined the neutron flux to be 7.25×10^{10} neutrons $cm^{-2}s^{-1}$. What value you deduce for the cross section for this reaction? Density of gold 19.30 gcm^{-3} [9.82b]

38. The radioisotope $^{15}O(T_{1/2} = 122s)$ is used to measure respiratory function. Patients inhale the gas by irradiating nitrogen gas with deuterons (^2H). Consider a cubical cell measuring 1.24 cm on each edge which holds nitrogen gas at a pressure of 2.25 atm and a temperature of 293K. One face of the cube is uniformly irradiated with a deuteron beam having current of 2.05A. At the chosen deutron energy, the reaction cross section is 0.21b. a) At what rate is ^{15}O produced in the cell? b) After an irradiation lasting for 60.0s, what is the activity of ^{15}O in the cell?   [a) $4.03 \times 10^8 s^{-1}$ b) 3.14m Ci]

39. 30 milligrams of gold are exposed to a neutron flux of 3.0×10^{12} neutrons $cm^{-2}s^{-1}$ for 1.0 minute. The neutron capture cross section of gold is 99b. Find the resultant activity of ^{198}Au ? $(130\mu Ci)$

40. Find the Q-value of the reaction

 $$p + {}^{55}Mn \rightarrow {}^{54}Fe + 2n$$
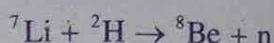
    $$m(^1H) = 1.007825u, \quad m(Mn) = 54.938045u,$$

    $$m(Fe) = 53.939614u, \quad m_n = 1.008665u$$

    $(-10.313MeV)$

41. Find the Q-value of the reaction

$$^7\text{Li} + {}^2\text{H} \rightarrow {}^8\text{Be} + n$$

$$m(\text{Li}) = 7.016005u, \quad m({}^2\text{H}) = 2.014102u,$$

$$m(\text{Be}) = 8.005305u, \text{ and } m_n = 1.008665u \qquad (15.032 \text{ MeV})$$

42. Calculate the threshold kinetic energy for the reaction $p + {}^3\text{H} \rightarrow {}^2\text{H} + {}^2\text{H}$

   a) If protons are incident on ${}^3\text{H}$ at rest

   b) If ${}^3\text{H}$ (tritons) are incident on protons at rest. $m({}^1\text{H}) = 1.007825u$,

   $m({}^3\text{H}) = 3.016049u$ and $m({}^2\text{H}) = 2.014102u$

   [a) 5.381 MeV b) 16.10 MeV]

43. Find the Q-value and therefore the energy released in the fission reaction
   $^{235}\text{U} + n \rightarrow {}^{93}\text{Rb} + {}^{141}\text{Cs} + 2n$. Use $m(\text{U}) = 235.043930u$, $m_n = 1.008665u$, $m(\text{Rb}) = 92.922402u$ and $m(\text{Cs}) = 140.9200464$. $\qquad (179.94 \text{ MeV})$

44. Find the energy difference between ^{235}U and ^{236}U. We can regard this as the excitation energy of ^{236}U. Repeat this for $^{238}\text{U} + n$ and ^{239}U. Comparing the two results and justify ^{235}U is fissionable with slow neutrons and ^{238}U requires fast neutrons.

45. In the D-T reaction, the kinetic energies of ^2H and ^3H are small compared with typical nuclear binding energies. Find the kinetic energy of the emitted neutron. Q-value is 17.6 MeV and $\dfrac{m_n}{m_{\text{He}}} = 0.25$ $\qquad (14.1 \text{ MeV})$

Section C
(Answer questions in about **one** *or* **two** *pages)*
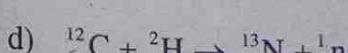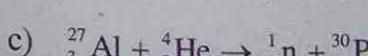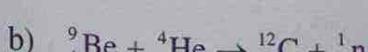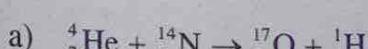
Long answer type questions (Essays)

1. Derive an expression for threshold kinetic energy of nuclear reaction?

2. Explain the construction and working of a nuclear fission reactor?

Hints to problems

34. Use conservation number of protons and neutrons on both sides of the reaction.

   a) ${}^4_2\text{He} + {}^{14}_7\text{N} \rightarrow {}^{17}_8\text{O} + {}^1_1\text{H}$

   b) ${}^9_4\text{Be} + {}^4_2\text{He} \rightarrow {}^{12}_6\text{C} + {}^1_0n$

   c) ${}^{27}_3\text{Al} + {}^4_2\text{He} \rightarrow {}^1_0n + {}^{30}_{15}\text{P}$

   d) ${}^{12}_6\text{C} + {}^2_1\text{H} \rightarrow {}^{13}_7\text{N} + {}^1_0n$

35. The energy levels are:
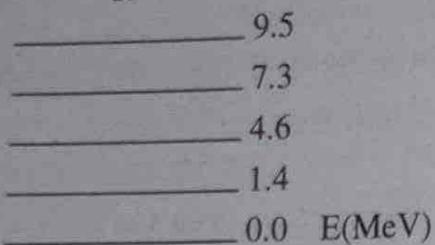
$$16.2 - 14.8 = 1.4 \, \text{MeV}$$
$$16.2 - 11.6 = 4.6 \, \text{MeV}$$
$$16.2 - 8.9 = 7.3 \, \text{MeV}$$
$$16.2 - 6.7 = 9.5 \, \text{MeV}$$

So the energy levels are

    ————————— 9.5

    ————————— 7.3

    ————————— 4.6

    ————————— 1.4

    ————————— 0.0   E(MeV)

36. Using $\quad R = \dfrac{N\sigma I_0}{S} = \dfrac{mN_A}{M}\dfrac{\sigma I_0}{S}$

but $\quad m = St\rho \quad \therefore \quad R = \dfrac{N_A I_0 \sigma t \rho}{M}$

$$I_0 = \frac{3.0 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.875 \times 10^{13} \text{ protons per second}$$

$$R = \frac{6.022 \times 10^{23} \times 1.875 \times 10^{13} \times 0.4 \times 10^{-24} \times 10^{-4} \times 7.9}{56}$$

$$R = 6.37 \times 10^7 \text{ particles per second.}$$

37. Using $\quad R = \dfrac{\sigma \phi m N_A}{M}$

$$\sigma = \frac{MR}{\phi m N_A} = \frac{197 \times 5.37 \times 10^6}{7.25 \times 10^{10} \times m \times 6.022 \times 10^{23}}$$

Where $\quad m = V\rho = \pi r^2 t \rho = 3.14 \times (0.15)^2 \times 1.81 \times 10^{-4} \times 19.3$

$$= 2.468 \times 10^{-4}$$

$$\sigma = 9.82 \text{ barn}$$

38. The number of nitrogen molecules in the cell is:

$$N = \frac{PV}{kT} = \frac{2.25 \times 1.01 \times 10^5 \times (0.0124)^3}{1.38 \times 10^{-23} \times 293}$$

$$N = 1.07 \times 10^{20}$$

The number nitrogen nuclei = 2N.

$$I_0 = \frac{2.05 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.28 \times 10^{13}$$

$$R = \frac{\sigma N I_0}{S}$$

$$\sigma = 0.21 \times 10^{-28} \, m^2 \quad N = 2.14 \times 10^{26} \quad \text{and} \quad s = 0.0124^2$$

Thus $\quad R = 4.03 \times 10^8 \, s^{-1}$

Activity $\quad a(t) = R = (1 - e^{-\lambda t}) = 1.16 \times 10^8 \, s^{-1} = 3.14 \, mCi$

39. $R = \phi \, \sigma \, \dfrac{m}{M} \, N_A = \dfrac{3 \times 10^{12} \times 99 \times 10^{-24} \times 0.03 \times 6.02 \times 10^{23}}{197}$

$R = 2.7 \times 10^{10} \, s^{-1}$

In the case $t \ll T_{1/2}$

$\therefore \; a(t) = R \lambda t = 4.8 \times 10^6 \, s^{-1} = 130 \mu Ci$

40. $Q = [m(^1H) + m(M_n) - m(F_e) - 2m_n]c^2$

$1u = 931.5 \dfrac{MeV}{c^2}$

$Q = -10.313 \, MeV$

41. $Q = \left[m(Li) + m(^2H) - m(Be) - m(n) \right] c^2$

$Q = -15.0316 \, MeV$

42. $Q = \left[m^1H + m(^3H) - 2m(^2H) \right] c^2 \quad = -4.0335 \, MeV$

a) $\quad K_{Th} = -Q \left[1 + \dfrac{m(^1H)}{m(^3H)} \right] = 3.381 \, MeV$
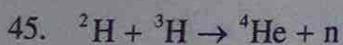
b) $\quad K_{Th} = -Q \left[1 + \dfrac{m(^3H)}{m(^1H)} \right] = 16.10 \, MeV$

43. $Q = \left[m(U) + m_n(n) - m(Rb) - 2m_n \right] c^2$

$\qquad = 179.94$ MeV

44. $^{235}U + n \rightarrow \, ^{236}U$

$\qquad \Delta E_9 = \left[m(^{235}U) + m_n - m(^{236}U) \right] c^2 = 6.546$ MeV

$\qquad ^{238}U + n \rightarrow \, ^{239}U$

$\qquad \Delta E_2 = \left[m(^{238}U) + m_n - m(^{239}U) \right] c^2$

$\qquad \Delta E_2 = 4.807$ MeV

The excitation energy of ^{235}U is greater than the excitation energy of ^{238}U. The second one needs an additional energy of $(6.546 - 4.807) = 1.739$ MeV. This should come from the kinetic energy of neutrons. So ^{238}U can be fissioned only by fast neutrons.

45. $^2H + \, ^3H \rightarrow \, ^4He + n$

$\qquad Q = K_{He} + K_n = 17.6 \text{MeV}$ given.

$$Q = \frac{P_{He}^2}{2m_{He}} + \frac{p_n^2}{2m_n}$$

Neglecting initial kinetic energies of ^2H and ^3H. Then conservation of momentum gives

$$O = P_{He} + P_n$$

or $\qquad P_H = - P_n$

$\therefore \qquad$
$$Q = \frac{p_n^2}{2m_{He}} + \frac{p_n^2}{2m_n}$$

$$Q = \frac{p_n^2}{2m_n} \left(1 + \frac{m_n}{2m_{He}} \right)$$

$\therefore \qquad$
$$Q = \frac{p_n^2}{2m_n} = \frac{Q}{1 + \dfrac{m_n}{m_{He}}} = \frac{17.6}{1 + 0.25}$$

$\qquad K_n = 14.1$ MeV

---

③ PARTICLE DETECTORS

Introduction

In this chapter we shall discuss nuclear radiation detectors and counters. The radiation could be charged particles, photons and neutrons. The interaction of any type of radiation always involves charged particles at some stage in the process. For uncharged radiations such as γ-rays and neutrons there is transfer of some of the energy to charged particles before there is any effect on the absorbing medium.

Nuclear detectors

The evolution of the techniques of nuclear radiation detection has played a vital role in unraveling the mysteries of the atomic nucleus. The radiations coming out of the nucleus such as the α, β and γ rays are the signals which carry with them information about the properties of the nucleus. Hence their detection and measurement are of prime importance in understanding the structure of the nucleus.

The principle of nuclear radiation detection can be broadly divided into three classes.

(a) Methods based on the detection of free charge carriers: During the passage of an ionizing radiation through a medium (solid, liquid or gas) both positive and negative ions are produced. Since ionizing radiation comprises charged particles moving with high velocity, the method is primarily applicable in the case of charged particle detection. Uncharged radiation like gamma rays or neutrons can also be detected by instruments based on this method since they usually eject charged particles which then cause ionization in the medium.

Instruments based on this method include ionisation chambers, proportional counters, Geiger-Muller counters and semi-conductor detectors.

(b) Methods based on light sensing: These are also applicable for both charged particle detection and detection of uncharged radiation.

Instruments based on this method include scintillation counters and Cerenkov detectors.

(c) Methods based on the visualization of the tracks of the radiation:

These are applicable for the detection of charged particles and include instru-

ments like the Wilson cloud chamber, bubble chamber, nuclear emulsion plates, spark chamber and solid state track detectors.

Hybrid detectors combining both ionization method and light sensing method have been used for special purposes.

Methods for the detection of free charged carriers

When an energetic heavy charged particle moves through a substance, it loses energy by repeated ionising collisions with the atomic electrons in the substance. In each collision, a pair of positive and negative ions is produced. The negative ions are usually electrons. In the case of a solid medium, instead of a positive ion, a positively charged hole is created. The positive and negative ions including the positive holes are the charge carriers. This ionisation process is known as primary ionisation. The positive and negative electrodes placed within a detector attract the oppositely charged ions produced in the medium between them which gives rise to an ionisation current. This can be recorded by a suitable measuring device to record the event (i.e., the passage of the particle through the medium) while moving towards the electrodes, the ions suffer repeated collisions with the atoms in the medium. If the potential difference between the electrodes in the detector is sufficiently high, then the primary ions produced in the medium may gain an amount of energy high enough to produce another ion pair which will also move towards the opposite electrodes. These in turn may produce further ionisation by collision. All the secondary ions thus produced add up with the primary ions and thus an amplified current is recorded by the detector. This process is known as gas amplification. Ionisation champers, proportional counters, Geiger Muller counters and semiconductors are based on this method, which we are supposed to study.

Wilson cloud chamber

It is an instrument used for the visual observation of the tracks of the charged particles in their passage through matter. C.T.R. Wilson was the person who first designed this and hence called Wilson cloud chamber.

Principle

When air mixed with saturated water vapour is suddenly expanded (adiabatic), it will result in fall of temperature and super saturation of vapour. The vapour will condense in cloud of water droplets. The production of such a cloud is however impossible unless nuclei on which the water vapour may condense are provided. If particles of dust are present in the air they will act as condensation nuclei. But in dust free air this phenomenon of condensation and formation of cloud will not occur. C.T.R Wilson discovered that the condensation cloud could be produced even in

dust free air provided the air was ionised by an ionising agent such as X-rays, cathode rays or any other nuclear radiations. When ionising agent entered the chamber immediately before or after the expansion, the ions left in its path would act as a condensation nuclei.

The experiment shows that as super saturation increases, the negative ions first serve as centres of condensation, then as the volume increases both positive and negative ions serve as the nuclei of droplets.

Figure 3.1

In short the working principle of cloud chamber is that charged ions can serve as nuclei of condensation of water droplets in a dust free atmosphere of air-water vapour mixture under the conditions of supersaturation.

Construction

Wilson cloud chamber consists of a large air tight cylindrical chamber A with the walls and ceilings are made of glass. The chamber contains dust free air saturated with water vapour or some other volatile liquid. The mixture inside the chamber can be compressed or expanded with the help of a piston p. The surface of the piston is usually covered with black felt so that no light is reflected from it to enter into the camera facing the front side of the chamber.

Working

To begin with the piston p is moved suddenly down. As a result of this the mixture undergoes adiabatic expansion due to which temperature falls. At this reduced temperature the mixture becomes supersaturated with the vapour and remains as such without any condensation of the vapour. For condensation to occur nuclei centres are required. For this a power full light flash is allowed to pass through the mixture immediately before or after moving the piston. This results in ionisation. The positive
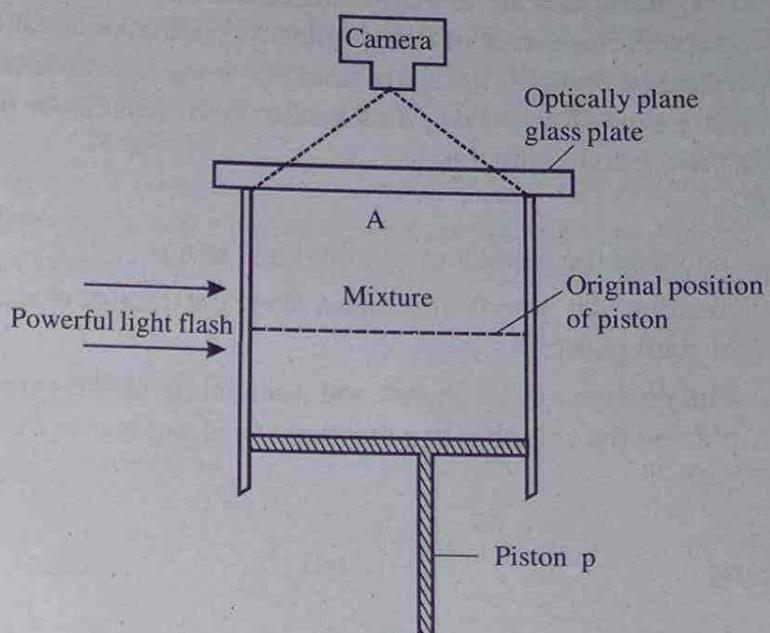
and negative ions act as centres of condensation i.e., the ions act as the nuclei of droplets. A close array of fine droplets i.e., a kind of linear cloud called a cloud track will thus be formed. By using suitable strong illumination from the side, the track appears as a white line on a dark background. This can be photographed by means of a camera fitted at the top.

Uses

1. To study the behaviour of individual atoms
2. To study the specific ionisation along the track of charged particles and the range of such particles.
3. The polarity of the charge and momentum of the particle can be determined by placing the chamber in a magnetic field and noting the radius of curvature of the path.

Using $$\frac{mv^2}{r} = qvB$$

or $$mv = qBr$$

Note

1. The main disadvantage of the cloud chamber is that it needs a definite time to recover after an expansion. Hence it is not possible to have a continuous record of events taking place in the chamber.
2. It cannot be used for detecting high energy particles since the interaction of energetic particles cannot be completely observed in the chamber.

Bubble chamber

It is a high energetic particle detector discovered by D.A. Glaser at the University of Michigan in 1952.

Principle

We know that normally the liquid boils with the evolution of bubbles of vapour at the boiling point. If the liquid is heated under a high pressure, its boiling point is raised, above the normal boiling point, without boiling taking place. A sudden release of pressure will leave the liquid in a superheated state and the liquid does not boil immediately but remains quite for some time. If a ionising particle is incident on the liquid just after releasing the pressure, the ions left in the track of a particle act as condensation centres for the formation of bubbles. This is the principle on which bubble chamber works.

Construction

It consists of a cylindrical chamber containing liquid. To super heat the liquid a mechanism, with bellows and expansion piston, is fitted with the cylinder. The top and bottom of the cylinder are provided with optically plane glass plates. The whole system is enclosed in a temperature control. Light from a flash lamp enters the cylinder through the glass at the bottom of the cylinder. A camera is fitted at the top of the cylinder to take inside photograph of the cylinder. The bubble chamber is used in conjunction with electromagnet for determining the sign of the charge and momenta of the particles.
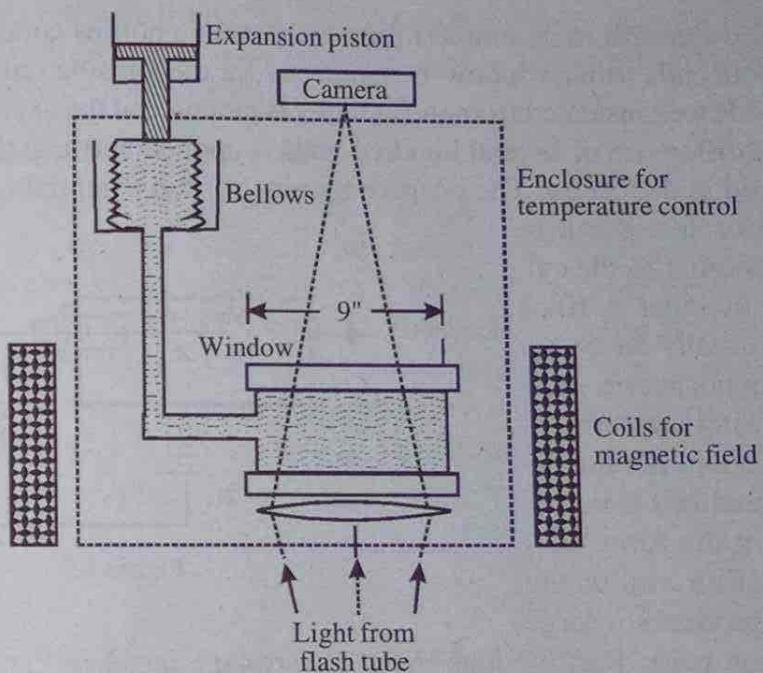
Figure 3.2

Working

The operating cycle of the bubble chamber consists of four stages.

1. The liquid is first heated to a temperature above its boiling point.
2. It is kept in the liquid state by the application of a pressure greater than its saturation vapour pressure.
3. The pressure is suddenly released so that the liquid becomes superheated and the flash lamp is switched on just before the pressure falls to minimum.
4. The grow of the bubbles occur for about milliseconds and the centres for the bubble formation last for about the same time.

A powerful lamp is then switched on within a millisecond to photograph the tracks. The cycle is then repeated.

Note : Various liquids have been used in the bubble chamber. Glaser used ethyl ether, later liquid hydrogen, deuterium and helium have been used. These have the advantage of having simple nuclei but the difficulty lies in their boiling points being very low. Thus the hydrogen chamber must be operated at about 26K. Heavy liquids such as pentane, propane and xenon have also been used.

Ionisation chamber

Ionisation chamber in its simplest form consists of a hollow conducting cylinder closed at both ends with a window W at one end for the ionising radiations to enter. A metal rod R well insulated from the cylinder is mounted at the axis of the cylinder. A potential difference of several hundred volts is applied between the cylinder and the metal rod at the centre. The positive terminal of the potential to the metal rod through a resistance R and the negative potential to the cylinder. The cylinder is filled with a gas usually air or hydrogen at atmospheric pressure or at greater pressures for γ – ray detection. For the detection of neutrons boron is introduced in the form BF_3. When radiation enters the chamber it produces a large

Figure 3.3

number of ion pairs. Positive ions move towards the metal rod at the centre and negative ions towards the metallic cylinder. In order to count particles the pulses of current produced are fed to an amplifier.

Proportional counter

When particles of low specific ionisation passes through an ionisation chamber, the pulses produced is too small to detect. One way of amplifying the signal obtained from a gas- filled detector is to increase the electric field so that the electrons gain enough energy between collision with gas atoms to cause further ionisation. In such a case the electrons produce further ionisation, and a rapid amplification of the original cascade occurs in what is called as Townsend avalanche. It should be noted that the size of the output signal is proportional to the number of ions formed by the primary ionisation process. Such a chamber is called a proportional counter.
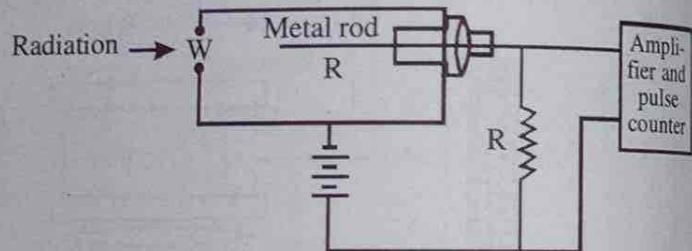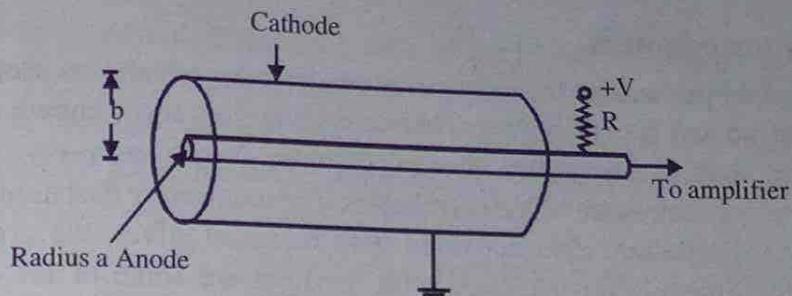
Figure 3.4

The proportional counter consists of a cylindrical gas filled tube (radius b Fig.) with a very thin central wire (radius a) which is insulated from the tube. The central wire is positive with respect to the tube which serves as the collecting electrode. This is connected to a pulse amplifier.

Geiger-Muller counter

Geiger-Muller Counter (G.M. counter) is in effect an improvement from the ionisation chamber. The principle of this instrument is the fact that the ions created by the entry of an elementary particle move in an intense electric field with such a high speed that ionisation by collision results. This causes a pulse of current which is used to record the incidence of the ionising particle. In G.M. counters this pulse is independent of the number of ions produced initially, while in proportional counter, the amplitude of the pulse is proportional to number of ions produced initially.

A G. M tube can be built in different geometries. Two most common forms are shown in Fig. (a) and (b)
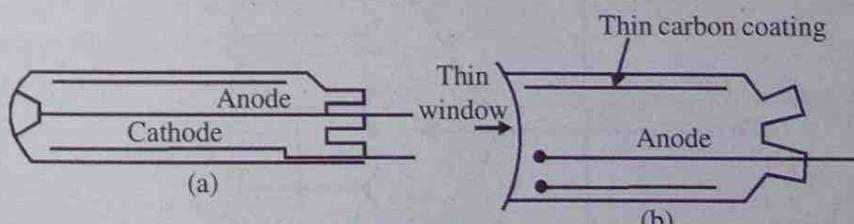
Figure 3.5

The first of these Fig (a) consists of a cylindrical copper cathode and a thin axial tungsten wire anode supported inside a cylindrical glass envelop. The ionising particle enters the counter through its glass walls. The second type Fig (b) is an end window counter. This modification is useful for less penetrating particles. The window is generally made of thin mica sheets. The anode is a tungsten wire and the cathode is in the form of a graphite coating.

Scintillation counters

This one of the earliest forms of radiation detectors which was used extensively by Rutherford and his colleagues. There are some substances known as phosphors which emit light flashes when charged particles, X-rays or γ-rays pass through them. These substances are called scintillators. The momentary flashes of light emitted are called scintillation. ZnS activated with traces of silver, the organic crystal anthracene stilbene, NaI activated with thallium are some of the examples of phosphors.

In older days the visible scintillations produced by the incident radiation were observed and counted through a microscope fitted with a screen coated with phosphor. This method of counting was very painstaking and required a period of adaptation of the eye within a dark room. ZnS was the phosphor used by Rutherford for the detection and counting of alpha particles.
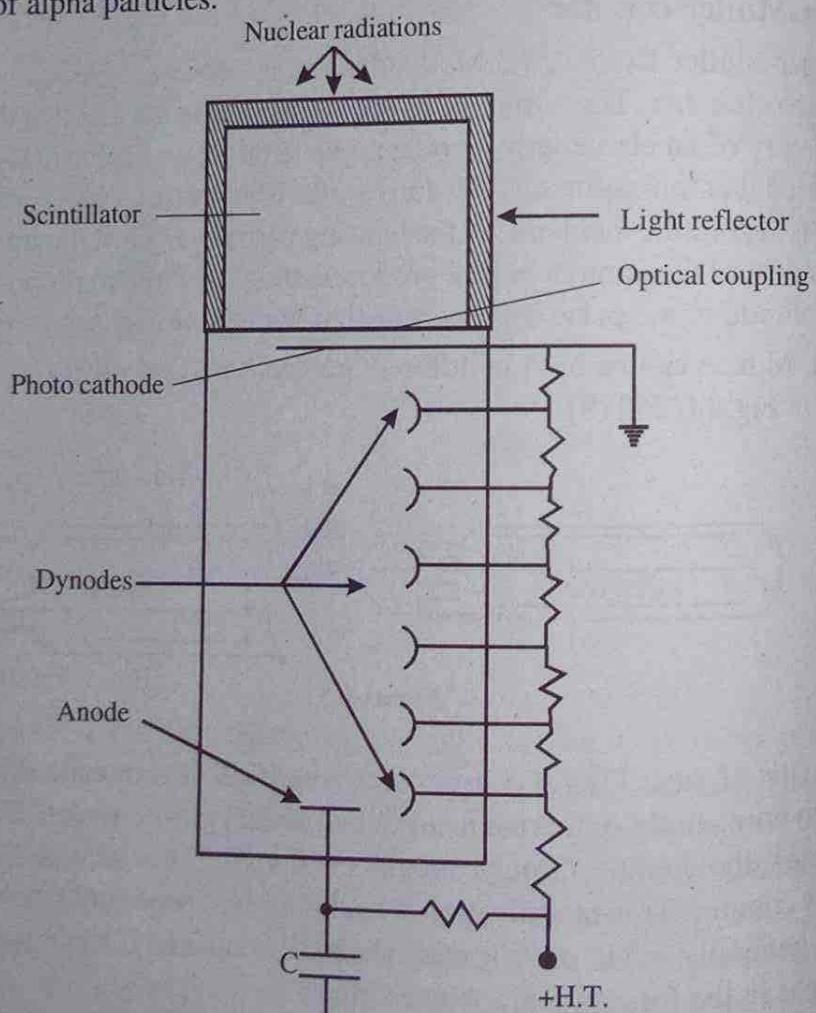
Figure 3.6

Nowadays very high speed electronic devices have been developed for detecting the scintillations taking place in times of the order of nano-seconds. Besides the detection of charged particles like α or β rays, high speed protons, deutrons can also be detected by the scintillation counters. This method has been found to be specially useful for γ-ray detection with high efficiency.

A schematic diagram of modern scintillation counter is shown below. A nuclear radiation falling on the scintillator can dissipate all its energy in it if the dimensions of the scintillator are large compared to its range. The scintillator produces scintillations (light pulses) which reach the photo cathode of a photo multiplier tube which is optically coupled to the scintillator. The photo multiplier tube consists of a photo cathode and several dynodes maintained at successively higher potentials of about 100 volts per dynode stage. When light pulses reach on photo cathode emit photoelectrons. A photo electron emitted by the photocathode is accelerated by the electric field to the first dynode where it produces a bunch of secondary electrons. These electrons are accelerated to the second dynode and produce more electrons. This process is repeated at each dynode. There are usually ten or more dynodes which ultimately achieve a gain of 10^7 to 10^8 by the time the electrons reach the last stage called the anode.

The magnitude of the output pulse from the photomultiplier is proportional to the energy of the particles incident upon the phoshor in a given scintillation counter. These pulses are fed to a pulse amplifier followed by a scaler circuit.

Advantages

1. It has a much higher counting efficiency for gamma rays due to the greater amount of energy dissipation by gamma rays.
2. It offers greater stability, greater accuracy, shorter resolving time and higher efficiency than G.M. counters.

Applications

1. Scintillation counters can be used to measure the particle energy.
2. Because of their rapid response they are used for the accurate timing of nuclear particles moving with very high speeds.
3. They are used to detect of antiprotons.
4. They are used in the study of cosmic rays, for the detection of mesons and other unstable particles of very high energy.

Semiconductor detectors

Semiconductor detectors are essentially reverse biased junction diodes. They have

several advantages over other types of radiation detectors. They have better energy resolution, linear response over a wide range of energies of the incident radiation and fast time response.

A semiconductor detector consists of a semiconducting crystal sandwitched between two conducting electrodes. An electric field is applied across the electrodes. Passage of nuclear radiation produces hole-electron pairs which should move towards the appropriate electrode and the charge collected is used as a signal of the detection of radiation.

Figure 3.7

There are two types of semiconductor detectors:

(i) Diffusion junction detector

(ii) Surface barrier detector.

Diffusion junction detector

In a p-n junction diode, the electrons from the n-region and the holes from the p-region diffuse into the p and n regions respectively across the junction and form a double layer of charges, which prevents further diffusion and a self-adjusted potential barrier is created at the junction. If now the p-region is connected to the negative terminal of a battery and the n-region to the positively terminal, the arrangement is said to be reverse-biased (see Fig). As a result, the electrons and holes are drawn away from the junction region and a depletion layer is formed at the junction in which no charge-carriers are present (see Fig). Due to reverse bias, no current flows through the junction and the diode is cut off. The thickness of the depletion layer depends on the nature of the impurities and on the voltage applied. It usually lies in the range of several hundred microns to a few millimetres. Since the conductivity of the depletion layer is low, a large potential difference can be maintained across it. So it acts like an ionization chamber.

If now a high energy charged particle passes through the depletion layer, electron - hole pairs are generated. The electrons are raised to the conduction band and are free to move through the crystal under the action of the applied electric field. The holes left behind in the valence band also move freely through the crystal . Due to both these reasons, a momentary current impulse proportional to the number of elec-
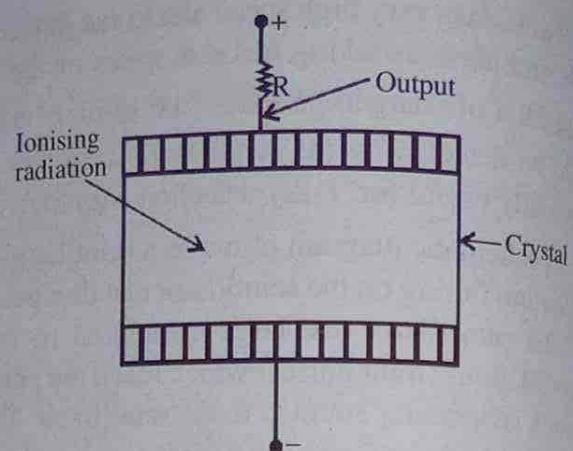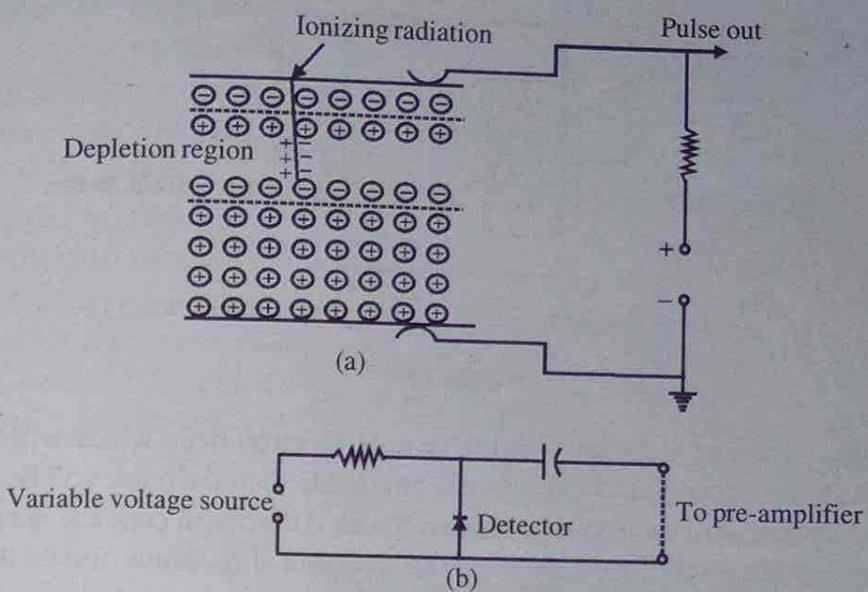
Figure 3.8

trons and holes is produced. Thus the semiconductor detector acts like a solid state ionization chamber. Since the energy needed to produce an electron-hole pair is only about 3 eV compared to about 30 eV required to produce an electron - ion pair in a gas ionization chamber, the registration of the electrical pulses in the semiconductor detector is easier and the accuracy of energy measurement much better (up to a fraction of a percent). In fact semiconductor detectors have the best energy resolution amongst all types of radiation detectors for most purposes. Because of the very narrow width of the depletion region, the pulse rise time is very small $(\sim 10^{-8}\,s)$ for these detectors.

Surface barrier detector

A very widely used surface barrier detector is the silicon surface barrier detector. A typical surface barrier detector is shown in figure.

It essentially consists of an extremely thin p-type produced on a high purity n-type silicon wafer. The combination constitutes a large area p-n junction diode. To construct the detector an n-type silicon wafer is taken and one of its surfaces is etched with acid and exposed to the air.

An oxidation layer is formed on the etched surface and this layer acts like a very thin p-type. Under the influence of an externally applied field electrons will be swept toward right and the holes to the left. As a result, an intermediate volume around the interface will be cleared of carriers of both signs and a depletion layer will be formed.

Ionising particle

Front (Au) electrode →

← Back (Al) electrode

n-type silicon wafer

Depletion layer
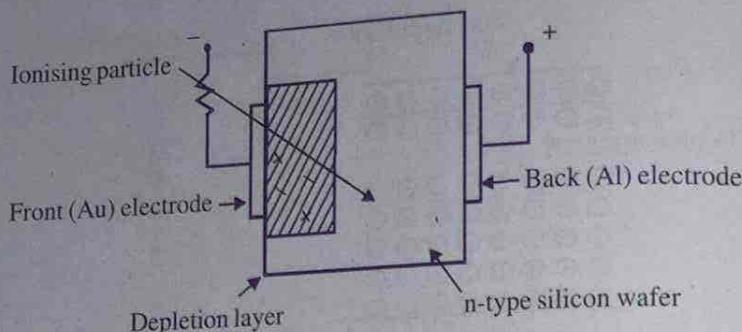
Figure 3.9

Passage of an ionising radiation will create hole-electron pairs which will be swept towards the appropriated electrodes by the applied field and a pulse will be obtained to signal the passage of the ionising radiation. Good electrical contacts are provided by evaporated thin gold film on the p-surface and thin aluminium film on the n-type silicon layer. These detectors have long stability, small size, negligible window absorption and linear pulse response over a wide range.

Spark chamber

This chamber is used in the field of high energy physics. It consists of a series of large parallel metal plates, several square feet in area, set in a chamber filled with neon gas at atmospheric pressure. All the plates are isolated from each other but alternate ones are grounded and the others are connected together to a high voltage d.c pulse generator (~ 15kV) which gives them high potential in short bursts of the order of a microsecond each. This is just enough to cause sparks to occur between the plates in such regions as are ionised by a particle entering the chamber. This gives a trail of sparks along the path of the particle which can then be photographed from the side.

Usually the plate separation is a few millimetres, a hundred or so,

Plate

Counter

Pulse generator

Clearing field

Coincidence circuit
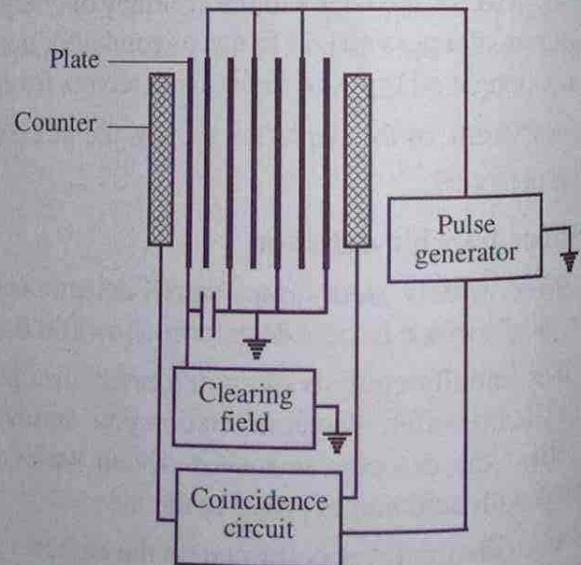
Figure 3.10: Schematic representation of spark-chamber circuit

these plates can make a volume of some several cubic feet. the schematic representation of the spark chamber is given in figure below.

One of the main advantages of the spark chamber over the bubble chamber is that triggering and removal of ions by the clearing field are comparatively simple. But the origin of an event can only be found within an accuracy of one plate thickness. Faster timing is also possible.

With suitable counters at the end of the spark chamber, the particular event expected can be made to trigger the high voltage generator and so record itself. Thus a selection of events to be studied may be made.

A photograph of spark chamber tracks is shown in figure 3.11(a).

Figure 3.11: (a) Spark-chamber tracks
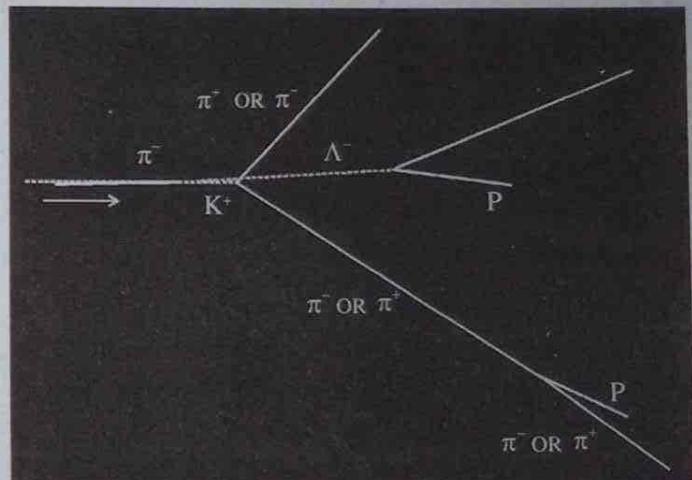
Figure 3.11: (b) Interpretation of tracks

Cerenkov counter

Cerenkov counters are particle detectors that make use of Cerenkov radiations.

When a charged particle passes through an optically transparent medium with a velocity greater than the phase velocity of light in that medium, it emits photons called Cerenkov radiation. The Cerenkov radiation is coherent at a particular direction (angle) relative to the direction of the particle.
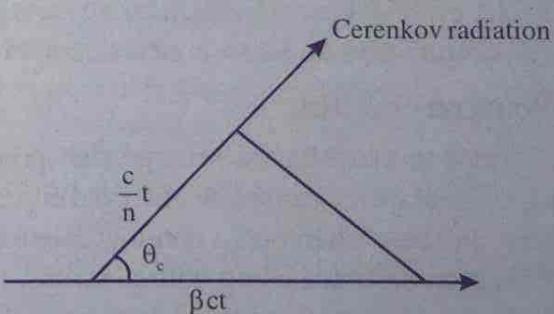
Figure 3.12

$$\therefore \qquad \cos\theta_c = \frac{c/nt}{\beta ct} = \frac{1}{\beta n}$$

The simplest Cerenkov counter consists of a barrel with enough volume containing distilled water to produced Cerenkov radiation. The outside of the barrel is silvered to make the inner side of the barrel reflective. This is to channel the light into a light gatherer. Just below it there is photon multiplier tube (PMT). PMT requires 1.5-2 kV for its operation. PMT is connected to a preamplifier. This is in turn connected to the output device. The schematic diagram of a Cerenkov counter is shown in figure 3.12.

Uses

1. It is used for prompt particle counting, the detection of fast particles, the measurement of particle masses etc.

2. It is used for tracking or localisation of events in very large natural radiators such as the atmosphere or natural ice fields like those of at the south pole in Antartica.

Figure 3.12: Cerenkov counter

3. Used as detectors in, high energy physics, particle accelerators, nuclear reactors, cosmic rays and also in neutrino astronomy.

Neutron counting

Since neutrons have no charge, they produce no paths of ions as they move through a gas. Hence they cannot be observed either in cloud chambers or in ionisation chambers. In these chambers counting depends upon ionisation. So we have to create ionisation particles along with neutrons. For example, if boron is bombarded with neutrons, α-particles are produced:

$$^{10}_{5}\text{B} + ^{1}_{0}\text{n} \longrightarrow ^{4}_{2}\text{He} + ^{7}_{3}\text{Li}$$

Here each neutron produces an alpha particle $\left(^{4}_{2}\text{He}\right)$. This alpha particle will produce ionisation track which can then be used to identify the neutron. Thus for a counter to detect neutrons it must contain some gas which ionises after neutron collision with it molecules. This is possible with BF_3 gas in which boron atoms produce the α-particles which in turn produce ionisation which can be detected in the usual manner. Neutron counting chambers are either ionisation or proportional counting arrangements.

The photographic plate

In particle and nuclear physics, nuclear emulsion plate is a photographic plate with a particularly thick emulsion layer with a very uniform grain size. Like bubble chambers and cloud chambers etc. nuclear emulsion plate records the tracks of charged particles passed through it. they are very compact and produce cumulative record.

The plates are darkened by radiation from radioactive substances and the darkening is due to the production of individual tracks. The photograph given below shows various dark lines revealed by the silver grains. In order to record these successfully, specially prepared plates are used with emulsion thickness of hundred microns. Each type of particle

Figure 3.13: Disintegration of an emulsion nucleus by a high energy proton. The proton enters the plate top centre and produces a star by collision with a silver or bromine nucleus.

has its own particular track. If neutrons are to be detected by this photographic plate, the plate must be socked in a boron solution.

A disadvantage of the nuclear emulsion plate is that, unlike the cloud chamber tracks, nuclear plate tracks cannot satisfactorily be bent in a magnetic field since large amount of scattering obscures the curvature of the tracks which, in any, case are very short.

Nuclear emulsion plates are very compact and cheap. Thus for most of the works in nuclear physics they go for these plates.

UNIVERSITY MODEL QUESTIONS

Section A

(Answer questions in **two** *or* **three** *sentences)*

Short answer type questions

1. What is nuclear radiation detector?
2. What is a cloud chamber?
3. What is an ionisation chamber?
4. How will you measure the momentum of the charged particles by using cloud chamber?
5. What is the disadvantage of cloud chamber?
6. Mention three applications of cloud chamber.
7. What is a bubble chamber?
8. Draw the schematic diagram of a bubble chamber and label it.
9. What is the principle of ionisation chamber?
10. What is a proportional counter?
11. What is a Geiger-Muller counter?
12. Why the scintillation counters called so?
13. Give three applications of scintillation counter.
14. What are the advantages of scintillation counters?
15. Draw a schematic diagram of a scintillation counter and label it.
16. What is a semiconductor detector?
17. Draw the schematic diagram of a spark chamber and label it.
18. What is Cerenkov radiation?
19. What is a Cerenkov counter?
20. What is a nuclear emulsion plate?

Section B

(Answer questions in a **paragraph** *of about half a page to one page)*

Paragraph / Problem type questions

1. Classify the principles of nuclear radiation detection.
2. What is the working principle of a cloud chamber?
3. What is the principle of bubble chamber?
4. Briefly explain an ionisation chamber.
5. What is the principle of proportional counter?
6. Briefly explain a Geiger-Muller counter.
7. Briefly explain surface barrier detector.
8. Briefly explain the function of a spark chamber.
9. What is the principle of Cerenkov counter?
10. Explain briefly the working of Cerenkov counter.
11. Explain briefly the "neutron counting".
12. Give three uses of Cerenkov counter.

Section C

(Answer questions in about **two** *pages)*

Long answer type questions (Essays)

1. Discuss the principle, construction and working of an ionisation chamber.
2. Explain the principle, construction and working of Wilson cloud chamber.
3. Discuss the principle, construction and working of bubble chamber.
4. Explain the principle, construction and working of a scintillation counter.
5. Explain diffusion junction detector in detail.

④ PARTICLE ACCELERATORS

Introduction

Particle accelerators are devices by which the charged particles can be energised. The accelerators played a vital role in the development of nuclear and particle physics. Before the advent of this machines, the only sources of high energy charged particles required for the study of nuclear transmutation were the naturally radio active substances emitting α and β particles. It is due to the limitations both of energy and intensity of the beam of particles the usefulness of these sources are limited. It was J.D. Cockcroft and E.T. S Walton, students of Rutherford, constructed a particle accelerator for the first time which would accelerate protons to high enough energy to produce nuclear transmutation.

Cockcroft - Walton proton accelerator

In 1932 Cockcroft and Walton designed and operated a proton accelerator at Cambridge. The basic principle used in this was that of a voltage doubler. The experimental arrangement is shown in figure below. It consists of a voltage doubler circuit, an accelerator to be provided with proton source.
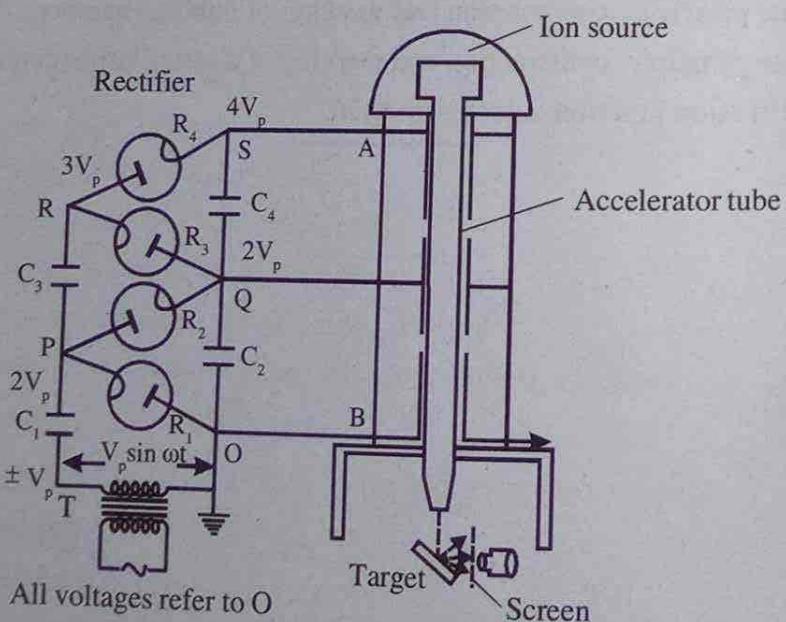

Figure 4.1: Schematic diagram of Cockcroft-Walton accelerator and discharge tube

Voltage doubler circuit

It is an electric circuit that generates a high voltage from low voltage. This voltage is used to power the accelerator. It is made up of a voltage multiplier ladder network of capacitors and rectifiers to generate voltage.

To understand the circuit operation, the circuit is powered by an alternating voltage V_i with a peak value of V_p and initially the capacitors are uncharged.

Switch on the input voltage V_i. When the input voltage V_i reaches its negative peak $(-V_p)$ current flows through the rectifier R_1 to charge the capacitor to a voltage V_p. When V_i reverses polarity and reaches its peak value (V_p), it adds to the capacitor voltage to produce a voltage of $2V_p$, the upper plate of C_1. Since R_1 is reverse biased current flows from C_1 through the rectifier R_2, charging the capacitor C_2 to a voltage of $2V_p$. When V_i reverses polarity again current from C_2 flows through the rectifier R_3, charging the capacitor also to voltage of $2V_p$. When V_i reverses polarity again current flows from C_3 through the rectifier R_4, charging capacitor to a voltage of $2V_p$.
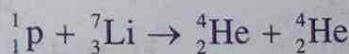
All the capacitors are charged to a voltage of $2V_p$ except C_1 which is charged to V_p. The key to the voltage doubler is that while the capacitors are charged in parallel, they are connected to the load in series. Since C_2 and C_4 are in series, between the output and ground, total output voltage is $4V_p$.

This circuit can be extended to any number of stages. The no load output voltage is twice the peak input voltage multiplied by the number of stages.

i.e., $$V_0 = 2NV_p = NV_{pp}$$

where V_{pp} is the peak to peak input voltage.

Cockcroft and Walton accelerator accelerated protons upto a voltage of 700 KeV. This accelerator performed the first nuclear disintegration by artificial means. They bombarded accelerated protons with lithium producing the reaction.

$$_1^1p + {}_3^7Li \rightarrow {}_2^4He + {}_2^4He$$

This was the first experiment to show that one element (Lithium) could be artificially transformed into another element. The Cockcroft-Walton accelerators are still widely used today. sometimes as injectors to much larger accelerators.

Note: Since the C.W accelerator produces only a low voltage proton reactions were limited to light elements like lithium, boron, beryllium etc.

Van de Graaff generator

A Van de Graaff generator is a device used for building up high potential differences of the order of several million volts. Such high potential differences are used to accelerate charged particles needed for various experiments of nuclear physics. It was designed by Robert J Van de Graaff in 1929.

Principle

This generator is based on

i)   The action of sharp points

ii) The property that charge given to a hollow conductor is transferred to outer surface and is distributed uniformly over it.

Construction

It consists of a large spherical conducting shell of radius of few metres. It is mounted on an insulating pillar of several metres high. An insulating belt is wound around the pullies P_1 and P_2. The belt is kept moving continuously over the pulley's with the help of a motor. B_1 and B_2 are two sharply pointed combs fixed as shown. B_1 is called spray comb and B_2 is called the collecting comb.

High voltage
electrode

Belt

Grounded

Insulator

Figure 4.2

Working

The spray comb is given a positive potential (-10^4 volts) with respect to earth by high tension source (H.T.). It is due to discharging action of sharp points a positively charged wind is set up, which sprays positive charge on the belt. As the belt moves and reaches the sphere a negative charge is induced on the sharp end of collecting comb B_1 and an equal positive charge is induced on the farther end of B_2. The positive charge shift immediately to the outer surface of sphere. Due to discharging action of sharp points of B_2 a negatively charged wind is set up. This neutralises the positive charge on the belt. The uncharged belt returns down and collects the positive charge from B_1 which in turn is collected by B_2. This is repeated. Thus the positive charge on the sphere goes on increasing. As $V = \dfrac{q}{C}$ and capacity of the spherical
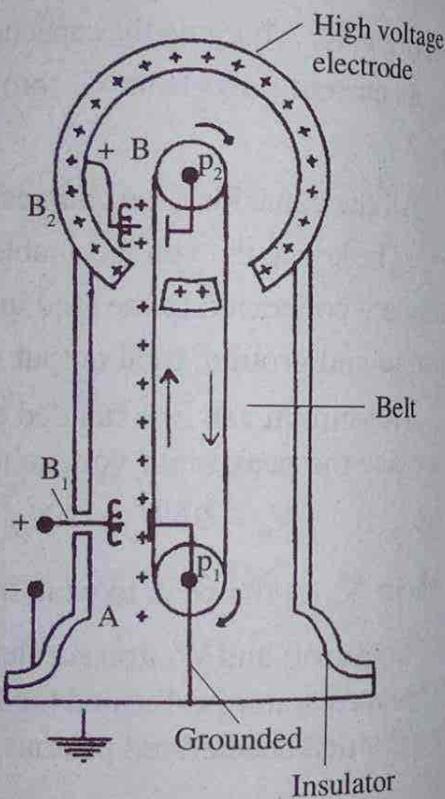
shell is constant, potential V of the shell goes on increasing till it reaches the desired value. With increase in V beyond a certain value leakage of charge to the surrounding occurs. This sets a limit on the maximum potential that can be achieved. The leakage is minimised by keeping the generator inside a steel chamber filled with nitrogen or methane at high pressure.

Linear accelerator (LINAC)

Linear accelerators (also called linac) accelerate charged particles along a straight line in multiple steps by an oscillating electric field. The first working linear accelerator was built by R. Wideroe in 1928 used only three coaxial cylindrical electrodes and could accelerate K^+ and Na^+ ions to twice the energy available for a single traversal of the field. In 1931, E.O. Lawrence and D.H. Sloan at the university of California Berkeley built an accelerator using 10 accelerating electrodes to produce 1.25 MeV mercury ions. Since then a large number of linear accelerators both for atomic ions and electrons have been built in different parts of the world. Here we discuss a simple form of linear accelerator.

Construction

It consists of a series of coaxial cylindrical drift tubes along the axis of which the charged particles travel. One set of alternate electrodes is connected to one terminal of the rf supply system while the other set of alternate electrodes is connected to the other terminal of the rf supply. In a linear accelerator for ions, the successive electrode tubes are of gradually increasing lengths. The cylindrical drift tubes are arranged inside a glass vacuum chamber. An ion source which is to be a accelerated is kept at A.
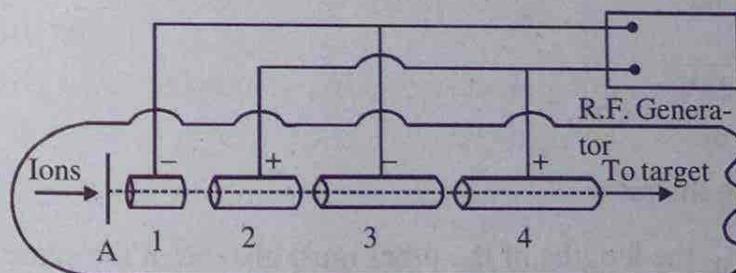
Figure 4.3

Principle of working

Suppose that the first drift tube is negative when the positive ion just leaves from A. On entering the tube the positive ion gets accelerated. The length of the first tube is adjusted in such way that the time taken by the positive ion to travel the entire length of the tube is exactly equal to the half the time period of the rf accelerating

voltage. When this condition is met the second drift tube becomes negative when the positive ion just leaves the first drift tube. As a result the positive ion again gets accelerated on entering the second tube. As the ions travel down the tube they do not gain any energy because of the screening action of the tube. i.e., ions get accelerated while crossing the gap.

If V be the potential of the first drift tube with respect to A, then the velocity v_1 of the ion on reacting the first drift tube is given by.

$$\frac{1}{2}mv_1^2 = qV$$

or $$v_1 = \sqrt{\frac{2qV}{m}}$$

where q is the charge of the ion and m is its mass.

While crossing the second gap between first drift tube and the second drift tube ions gain an additional amount of energy qV. If v_2 be the velocity of the ion on entering the second drift tube, then

$$\frac{1}{2}mv_2^2 = 2qV$$

or $$v_2 = \sqrt{\frac{4qV}{m}} = \sqrt{2}v_1$$

The process continues and when the ion crosses the nth gap, we have

$$\frac{1}{2}mv_n^2 = nqV$$

or $$v_n = \sqrt{\frac{n2qV}{m}} = \sqrt{n}v_1$$

since velocities are in the ratio

$1 : \sqrt{2} : \sqrt{3} : \ldots\ldots : \sqrt{n}$ the lengths of the tubes must also be in the same ratio.

It may also be noted that the final energy of the ions depends on the total number of gaps and the energy gained in each step.

Advantages of linear accelerators

(i)　It produces a relatively high beam current

(ii)　Extraction and injection of the high energetic beam is simple.

(iii)　Since they travel in straight line, it has practically no radiation loss.

The limitation of linear accelerator is that the length of the accelerator is inconveniently large and it is very difficult to maintain vacuum in such a large chamber.

Linear accelerators are used for electron acceleration, proton acceleration, ion acceleration, etc. They are used as injectors for high energy machines such as proton synchrotrons.

Cyclotron

A cyclotron is a device developed by Lawrence and Living Stone in 1931 by which the positively charged particles like proton, deuteron, α particle etc. can be accelerated.

Principle

A charged particle moving perpendicular in a magnetic field, it describes a circular path. It can be accelerated by allowing it to pass through a suitably adjusted electric field.

Construction

It consists of two D-shaped hollow evacuated metal chambers D_1 and D_2 called dees. These dees are placed horizontally with their diametric edges parallel and slightly separated from each other. The dees are connected to high frequency oscillator which can produce a potential difference of the order 10^4 V at $\approx 10^7$ Hz frequency. The two dees are enclosed in a steel box and well insulated from it. The box is placed in a strong magnetic field produced by two pole pieces of strong electromagnet NS. The magnetic field is perpendicular to the plane of the dees. At P the positively charged particles which is to be accelerated is kept.

Working and theory

The positively charged particle to be accelerated is produced at P. Suppose at that instant D_1 is at negative potential and D_2 is at positive potential. Therefore the particle will be accelerated towards D_1. On reaching inside D_1 it moves with a constant speed v in a circular path of radius r.

$$\therefore \qquad Bqv = \frac{mv^2}{r} \qquad \text{or} \quad r = \frac{mv}{qB}$$

Time taken by the particle to describe a semi circular path is given by

$$t = \frac{\pi r}{v} = \frac{m\pi}{qB}$$
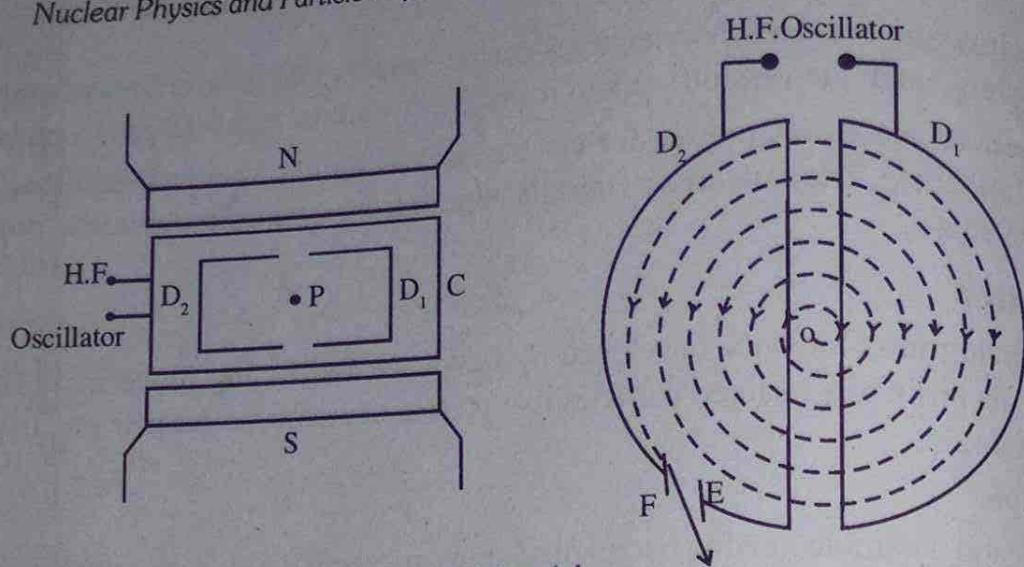
Figure 4.4

This time is independent of both the speed of the ion and radius of the circular path. In case the time during which the particle describes a semicircular path is equal to the time during which half cycle of oscillator is completed, then as the particle arrives in the gap between the dees, the polarity of the dees is reversed i.e., D_1 becomes positive and D_2 negative. Therefore particle is accelerated towards D_2 and it enters D_2 with greater speed which remains constant in D_2. The particle describe a semicircular path of greater radius. The process repeats and the particle goes on accelerating every time and acquires high energy. The accelerated particle can be removed out of the dees by applying the electric field across the deflecting plates E and F.

Maximum energy of the particle

Let v_0 be the maximum velocity and r_0 the radius of the circular path.

Then
$$\frac{mv_0^2}{r_0} = qBv_0$$

or
$$v_0 = \frac{qBr_0}{m}$$

$\therefore$ Maximum $K.E = \frac{1}{2}mv_0^2 = B^2q^2\frac{r_0^2}{2m}$

Cyclotron Frequency

If T is the time period of oscillating electric field then

$$T = 2t = \frac{2\pi m}{qB}$$

the cyclotron frequency is given by

$$\upsilon = \frac{1}{T} = \frac{qB}{2\pi m}$$

One limitation of cyclotron is that, it cannot accelerate a charged particle beyond a limit. This is because when the velocity of a charged particle increases its mass varies according to the equation $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$. As a result the resonance frequency also varies. This happens when v is comparable with c. The particles take longer time to describe semicircle. And it may become out of step with the accelerating voltage. That is instead of accelerating it may be retarded.

Note : Lawrence and living stone won the Nobel prize in 1934 for the construction of cyclotron.

Example 1

A cycloron is connected to the oscillator of frequency 15 MHz. What should be the operating magnetic field for accelerating protons. The radius of the dees is 60cm. Calculate the maximum kinetic energy of the proton in eV, the mass of the proton $= 1.67 \times 10^{-27}$ kg. $1eV = 1.6 \times 10^{-19}$ J

Solution

$$\upsilon = 15MHz = 15 \times 10^6 \, Hz$$

$$m = 1.67 \times 10^{-27} \, kg \ .q = 1.6 \times 10^{-19} \, C$$

we have $\upsilon = \dfrac{qB}{2\pi m}$

or $\quad B = \dfrac{2\pi m \upsilon}{q}$

$$B = \frac{2 \times 3.14 \times 1.67 \times 10^{-27} \times 15 \times 10^6}{1.6 \times 10^{-19}} = 0.99 \, T$$

Maximum $K.E = \dfrac{B^2 q^2 r_0^2}{2m}$

$$= \frac{(0.99)^2 \times (1.6 \times 10^{-19})^2 \times (0.6)^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19}} eV$$

$$= 16.9 \times 10^6 eV = 16.9 MeV$$

Example 2

An electron after being accelerated through a p d of 100V enters a uniform magnetic field of 0.004 T perpendicular to its direction. Calculate the radius of the path described by the electron.

Solution

V = 100 volt, B = 0.004 T

Let v be the velocity acquired by electron when accelerated under a p d, we have

$$\frac{1}{2} mv^2 = eV$$

or $$v = \sqrt{\frac{2eV}{m}}$$

we have $$r = \frac{mv}{qB} = \frac{\sqrt{2meV}}{qB}$$

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 100}}{1.6 \times 10^{-19} \times 0.004}$$

$$= 8.4 \times 10^{-3} m$$

Example 3

The electric field in a cyclotron is reversed every $9.372 \times 10^{-8} s$. It is used to accelerate deuterons, each of mass 3.34×10^{-27} kg and charge $1.6 \times 10^{-19} C$. Calculate the flux density of the magnetic field.

Solution

Time period of the electric field, $T = 2 \times 9.372 \times 10^{-8}$

$$= 18.744 \times 10^{-8} s$$

Using $$T = \frac{2\pi m}{qB}$$

$$\therefore \quad B = \frac{2\pi m}{qT} = \frac{2 \times 3.14 \times 3.34 \times 10^{-27}}{1.6 \times 10^{-19} \times 18.744 \times 10^{-8}}$$

$$= 0.6993 \, Wb/m^2$$

Example 4

Deuterons are accelerated in a fixed frequency cyclotron to a maximum dee orbit radius of 0.88m. The magnetic flux density across the dees has a mean value of 1.4T. Calculate energy of the emerging beam and the frequency of the dee voltage.

$m = 3.34 \times 10^{-27} \, kg$

Solution

$$q = 1.6 \times 10^{-19} C, \quad r_o = 0.88m, \quad B = 1.4T$$

$$m = 3.34 \times 10^{-27} kg$$

Energy of the emergent beam

$$= \frac{B^2 q^2 r_0^2}{2m} = \frac{1.4^2 \times (1.6 \times 10^{-19})^2 \times 0.88^2}{2 \times 3.34 \times 10^{-27}} = 5.816 \times 10^{-12} J$$

Frequency of the voltage, $\upsilon = \dfrac{qB}{2\pi m}$

$$\upsilon = \frac{1.6 \times 10^{-19} \times 1.4}{2 \times 3.14 \times 3.34 \times 10^{-27}} = 1.067 \times 10^7 Hz$$

$$= 10.67 MHz.$$

Example 5

In a linear accelerator, proton accelerated thrice by a potential of 40kV leaves a tube and enters an accelerating space of length 30cm before entering the next tube. Calculate the frequency of the r.f. voltage
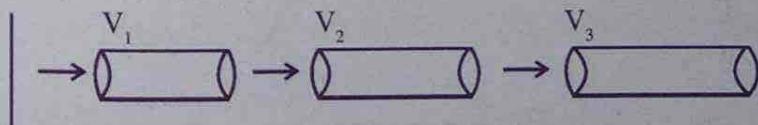
Solution

Figure 4.5

Let v_1 be the velocity of the proton on reaching the first drift tube, given by

$$\frac{1}{2}mv_1^2 = eV$$

or $$v_1 = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 40 \times 10^3}{1.67 \times 10^{-27}}} = 2.768 \times 10^6 \, ms^{-1}$$

while crossing the second gap between first drift tube and the second drift tube protons gain an additional amount of energy eV. If v_2 be the velocity of the protons entering the second drift tube, then

$$\frac{1}{2}mv_2^2 = 2eV$$

$$v_2 = \sqrt{2}\, v_1$$

Similarly on entering the third tube v_3 be the velocity which is given by

$$v_3 = \sqrt{3}\, v_1$$

Time taken to cover the third gap (0.30m)

$$t = \frac{dis\tan ce}{average\ velocity}$$

$$= \frac{0.3}{\left(\dfrac{v_2 + v_3}{2}\right)} = \frac{0.6}{v_2 + v_3} = \frac{0.6}{\sqrt{2}\, v_1 + \sqrt{3}\, v_1}$$

$$t = \frac{0.6}{(\sqrt{2} + \sqrt{3})v_1} = \frac{0.6}{(\sqrt{2} + \sqrt{3}) \times 2.768 \times 10^6}$$

This time is equal to the half the period $\left(\dfrac{T}{2}\right)$ of the r.f voltage

i.e., $$t = \frac{T}{2}$$

$$\frac{1}{T} = \frac{1}{2t} = \frac{(\sqrt{2} + \sqrt{3}) \times 2.768 \times 10^6}{2 \times 0.6}$$

$$= 7.257 \times 10^6 \, Hz$$

Synchrocyclotron

In the case of cyclotron the expression we used for the kinetic energy was the non-relativistic value $\frac{1}{2}mv^2$. For the early cyclotrons, working at low velocities, the non-relativistic value was enough. But for higher speeds relativistic effects will come into play.

For example if $v = 0.8c$, $\frac{v^2}{c^2} = 0.64$. So the mass changes according to the relation

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} = \frac{m_0}{\sqrt{1-0.64}} = \frac{m_0}{0.6} = 1.66m_0$$

In this situation the frequency of oscillation $\upsilon = \frac{qB}{2\pi m}$ will change. As m increases, υ decreases. This will upset the resonance condition. Therefore gradually the particle gets out of phase with the frequency potential on the dees. The frequency on the dees must therefore be compensated for the gain in mass, this is carried out by a rotating variable condenser giving the imposed frequency modulation required. Ions can then be accelerated to very high velocities, and the cyclotron becomes a synchrocyclotron. The synchrocyclotron in the Lawrence radiation laboratory produced 380 MeV alpha particles and was later redesigned to give 720 MeV protons.

The difference between the cyclotron and the synchrotron is that in the former the output is continuous but in the case of latter it is in bursts lasting about 100 microseconds.

Electron accelerating machines

The betatron

The device cyclotron can accelerate protons, deutrons, α – particles to very high energies of the order of several million electron volts. It doesn't require excessive high voltage for producing high energy particles. But as stated earlier it has a very serious limitation arising from relativistic variation of mass with energy for very high energy particles. When the relativistic variation of mass comes into play for high velocities, the frequency of revolution of the particle (cyclotron frequency) becomes

$$\upsilon = \frac{qB}{2\pi m} = \frac{qBc^2}{2\pi mc^2} = \frac{qBc^2}{2\pi(m_0c^2 + E_k)}$$

$$\therefore \quad mc^2 = m_0c^2 + E_k$$

where m_0 is the rest mass of the particle and E_k its kinetic energy when the velocity is v.

When the speed of the particle increases, υ and the spiralling particle lags in phase behind the accelerating voltage. Hence the acceleration will stop. The relativistic mass increase with thus limits the maximum energy attainable for a particle. In the case of electron the relativistic mass increase over the rest mass is considerable, since being light, even for moderate energies of electron (>10 keV). Hence resonance condition fails. This shows that electron is impracticable to be accelerated by cyclotron. To overcome this difficulty D.W. Kerst at the university of Illinois in the U.S.A devised a new instrument known as betatron in 1940 for the acceleration of electrons which based on an altogether new principle.

To overcome the relativistic effect we have to synchronies the revolution of the particles with the alternations of the accelerating p.d applied. This can be done in two ways.

1. By reducing the frequency of the alternating p.d applied which is made equal to the revolution frequency of the particles. Remember that the decrease in frequency of the particles is brought by the relativistic effect.

2. By increasing the magnetic field as the particle accelerate so as to keep the particles in phase with constant frequency of the alternating p.d applied.

The second method is adopted in the case of betatron and a combination of both for synchrotron.

Principle of betatron

The principle on which the action of a betatron depends is that the electrons moving in a stable orbit of fixed radius are accelerated by increasing the magnetic flux through the orbit.

Let the radius of the orbit be r and ϕ the total magnetic flux through the orbit, while the flux density is B in a direction perpendicular to the plane of the orbit. The stable orbit must be such that the average magnetic field over the space enclosed by the orbit is twice the magnetic field at the orbit.

Proof

If the total flux ϕ is increased at the rate $\dfrac{d\phi}{dt}$, the induced emf is given by

$$V = -\frac{d\phi}{dt}$$

Let $\bar{B}$ be the average value of the magnetic flux density within the area enclosed by the stable orbit of radius r

Thus $\phi = B \cdot \pi r^2$ $\left(\because \quad \dfrac{\phi}{A} = B \right)$

The work done on the electron in one revolution is therefore

$$-eV = 2\pi r \ F$$

where F is tangential force on the electron

or $F = \dfrac{-eV}{2\pi r} = \dfrac{e}{2\pi r} \dfrac{d\phi}{dt}$ ..... (1)

For the stability of the orbit, we must have

$$Bev = \dfrac{mv^2}{r}$$

or $mv = Ber$

Using $F = \dfrac{dp}{dt} = \dfrac{d}{dt}(Ber) = er \dfrac{dB}{dt}$ ..... (2)

It is due to increase in energy, the electron tends to move into an orbit of larger radius. If stability is to be maintained, this tendency is to be resisted. For this to happen equations 1 and 2 must be equal.

i.e., $\dfrac{e}{2\pi r} \dfrac{d\phi}{dt} = er \dfrac{dB}{dt}$

or $\dfrac{d\phi}{dt} = 2\pi r^2 \dfrac{dB}{dt}$ ..... (3)

This is known as the betatron condition. It shows that the electrons will revolve in an orbit of constant radius r, provided the magnetic flux ϕ changes at twice the rate at which it would change if the magnetic field had been uniform throughout the area enclosed by the electron orbit.

Integrating equation (3) gives

$$\phi_2 - \phi_1 = 2\pi r^2 B$$

assuming that the magnetic induction increases from 0 to B at the electron trajectory.

For a uniform magnetic induction, on the other hand, the flux would be $\pi r^2 B$. So for the betatron condition to hold, the magnetic flux be twice that for a uniform magnetic induction.

Construction

Betatron consists of a highly evacuated annular tube either made of glass or ceramics called doughnut. The doughnut tube is placed between specially shaped pole pieces of an electromagnet energised by passing alternating current through exciting coils. This a.c is supplied from the normal so cycles/s in which the increasing magnetic flux is obtained for quarter cycles i.e., for $\frac{1}{200}$ s. As a consequence electrons are accelerated for periods of $\frac{1}{200}$ s and out put beam is obtained in pulses. The pole pieces of the magnet are made of laminated iron to reduce eddy current losses and they have been shaped

Figure 4.6

such that the total flux through the orbit is twice the value that would have been if the field were uniform.

The electrons are emitted from a heated filament are first accelerated to about 50,000 volts and then injected into the doughnut at the instant when B = 0. The magnetic field then rises sinusoidally. The acceleration takes place during the first quarter of the time period T of the oscillating magnetic field. Exactly at the end of the acceleration period $\frac{T}{4}$ when B attains the peak value, an instantaneous current is passed through an auxillary coil which changes the magnetic field from the stable orbit value, as a result of which the electrons are deflected from stable orbit to hit a suitably located target, which makes the target emit x-rays within spread of energy from 0 upto the maximum electron energy by bremsstrahlung process. The process repeats itself at the interval of the time period T of the magnetic field so that the x-ray beam is obtained in pulses of very short duration during each interval T.

Energy of the electrons

Let the variation of the magnetic flux be $\phi = \phi_0 \sin \omega t$
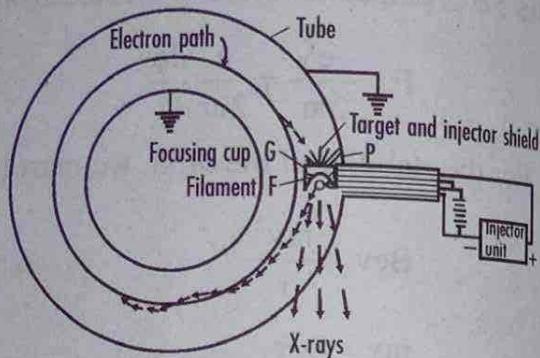
Energy per turn $= eV = e\dfrac{d\phi}{dt}$

$$= e\dfrac{d}{dt}(\phi_0 \sin \omega t)$$

$$= e\phi_0 \, \omega \cos \omega t$$

$\therefore$ Average energy per turn

$$\langle E \rangle = \int_0^{T/4} \dfrac{e\omega\phi_0 \cos \omega t \, dt}{\int_0^{T/4} dt}$$

Remember that the acceleration of the electrons takes places for quarter cycle.

$$\langle E \rangle = e\omega\phi_0 \int_0^{T/4} \dfrac{\cos \omega t \, dt}{\int_0^{T/4} dt}$$

$$\langle E \rangle = e\omega\phi_0 \dfrac{4}{T} \int_0^{T/4} \cos \omega t \, dt$$

$$\langle E \rangle = \dfrac{e\omega\phi_0}{\omega} \dfrac{4}{T} [\sin \omega t]_0^{T/4}$$

$$\langle E \rangle = e\phi_0 \dfrac{4}{T} \left[\sin \dfrac{2\pi}{T} \cdot \dfrac{T}{4} - 0 \right]$$

$$= e\phi_0 \cdot \dfrac{4}{T} = \dfrac{e\phi_0 4}{2\pi/\omega}$$

$$= \dfrac{2 e\phi_0 \omega}{\pi}.$$

Since the speed of the electron in the orbit is almost constant ($\approx c$) the total path travelled by the electron during acceleration is

$$L = c \cdot \dfrac{T}{4} \qquad\qquad (x = vt)$$

$$L = \dfrac{c \cdot 2\pi}{4\omega} = \dfrac{c\pi}{2\omega}$$

∴ The number of turns described by the electron

$$N = \frac{L}{2\pi r} = \frac{c\pi}{2\omega \cdot 2\pi r} = \frac{c}{4\omega r}$$

Hence final energy of the electron

$$= \frac{N \cdot 2e\omega}{\pi}\phi_0 = \frac{ec\ \phi_0}{2\pi r}$$

i.e., the final energy is determined by the peak value ϕ_0 of the magnetic flux and the radius of the orbit r.

For example $r = 0.5m$ and $B = 1T$

∴ $$\phi_0 = 2\pi r^2 B = 2 \times \pi \times 0.5^2 \times 1$$

$$= 1.57 \text{ weber.}$$

∴ Total energy $E = \dfrac{ec\phi_0}{2\pi r}$

$$E = \frac{1.6 \times 10^{-19} \times 3 \times 10^8 \times 1.57}{2 \times 3.14 \times 0.5} = 2.4 \times 10^{-11}\text{J}$$

or $$E = \frac{2.4 \times 10^{-11}}{1.6 \times 10^{-19}} = 150 \times 10^6 \text{eV} = 150 \text{ MeV}.$$

Note : In the first machine built by Kerst an electron beam of 2.3 MeV energy was obtained. One of the largest machine built at the Illinois university used a magnet weighing 356 tonne and the radius of the accelerator tube was 2.46m and obtained an energy of 340 MeV.

Example 6

The maximum magnetic induction in a betatron is 0.5T. If the radius of the dough-nut is 0.8m and the frequency of variation of the magnetic field is 50Hz. What is the energy gained per turn. What is the maximum energy of the electron

Solution

$$B = 0.5J, \quad r = 0.8m \quad \upsilon = 50Hz$$

Energy gained per t urn $= \dfrac{2e\phi_0\omega}{\pi}$

$$= \frac{2e\,2\pi r^2 B\,\omega}{\pi} = \frac{4\pi r^2 e\,B\,\omega}{\pi}$$

$$= 4er^2 B \cdot 2\pi\upsilon$$

$$= 4 \times 1.6 \times 10^{-19} \times 0.8^2 \times 0.5 \times 2 \times 3.14 \times 50\,J$$

$$= \frac{4 \times 1.6 \times 10^{-19} \times 0.8^2 \times 0.5 \times 2 \times 3.14 \times 50}{1.6 \times 10^{-19}}\,eV$$

$$= 4 \times 0.8^2 \times 0.5 \times 2 \times 3.14 \times 50\,eV$$

$$= 401.92\,eV .$$

Total number of revolutions

$$= \frac{c \cdot \dfrac{T}{4}}{2\pi r} = \frac{c \cdot 2\pi}{4\omega \cdot 2\pi r}$$

$$N = \frac{c}{4\omega r} = \frac{3 \times 10^8}{4 \times 2\pi \times 50 \times 0.8} = 2.985 \times 10^5$$

$\therefore$ Maximum energy $= N \times 401.92\,eV$

$$= 2.985 \times 10^5 \times 401.92\,eV$$

$$= 119.97 \times 10^6\,eV$$

$$= 119.97\,MeV .$$

Use of betatron

1. The X-rays produced by the electrons accelerated in the betatron are used to produce nuclear reactions, such as the (γ, n), (γ, p), (γ, 2n), and (γ, n, p) reactions, or as highly penetrating radiations for the study of the properties of solids.

2. The high energy electrons are used for research in biological sciences.

3. The x-rays are used for the treatment of cancer and other diseases. Many hospitals in different parts of the world use this facility now a days. In India one such machine has been installed in christian missionary hospital at Vellore in Tamilnadu.

Electron synchrotron

Synchrotron is an accelerator which can accelerate charged particles giving energy of the order of several hundred million electron volts. Synchrotrons are of two types, electron synchrotron and proton synchrotron. Here we shall discuss only about electron synchrotron. By adapting the betatron to use the synchrotron principle, Goward and Barnes in 1946 in England developed a machine which is known as electron synchrotron. The name synchrotron was suggested by Mac Millan because the behaviour of the machine is similar in some respects to that of a synchronous motor.

Principle of electron synchrotron

In the synchrotron the frequency of the rf oscillator is matched with the revolution frequency of the electron. The frequency of the rf field is kept unchanged and the matching of the frequencies being achieved by changing the magnetic field with time.

The initial acceleration in the synchrotron is produced by betatron principle. The changing magnetic field through the orbit of the electrons induces an electric field along the orbit which accelerates the electrons upto about 2 MeV. At this point the betatron action ceases and electrons revolve in the orbit with almost constant velocity $v \approx c$ so the angular speed $\omega = \dfrac{c}{r}$ is also constant and hence the orbit radius r remains practically constant.

The electrons are injected at fairly at high speed from the electron gun so that the orbit radius increases only marginally afterwards. After the initial betatron acceleration, the electrons revolve in stable orbits with angular speed $\omega = \dfrac{Be}{m}$, where m is the relativistic mass and gain energy due to acceleration by the rf field within the accelerator tube. Since m increases with the gain of energy, B must be increased to maintain synchronism.

Construction

Synchrotron consists of a doughnut (a ring shaped accelerator tube of toroidal section made of glass or porcelain) in an a.c magnetic field. The magnet can be a ring of C-shaped units not filling the hole in the doughnut as the case with the betatron. In the central gap of the doughnut some flux bars serve as the central core of the magnet to start up the machine as betatron. These bars are made of high permeability metal and donot have to be large, they short the magnetic field at low injections but become saturated at high inductions and the transition from betatron action

to synchrotron action can be made smoothly. Part of the interior of the doughnut is coated with copper or silver to given resonance cavity. A small break in the coating separates it into two parts. A high frequency electric field from a radio frequency oscillator is applied across this gap at the proper time in the magnetic cycle. When the accelerator is on, the electron is accelerated each time it crosses through the resonator.

The electron synchrotron accelerates electron in an orbit of constant radius by means of a radio frequency electric field applied across the gap. A ring shaped magnet provides the magnetic field over the doughnut shaped vacuum chamber. The pole faces are accurately shaped to provide a field which decreases with increasing radius.
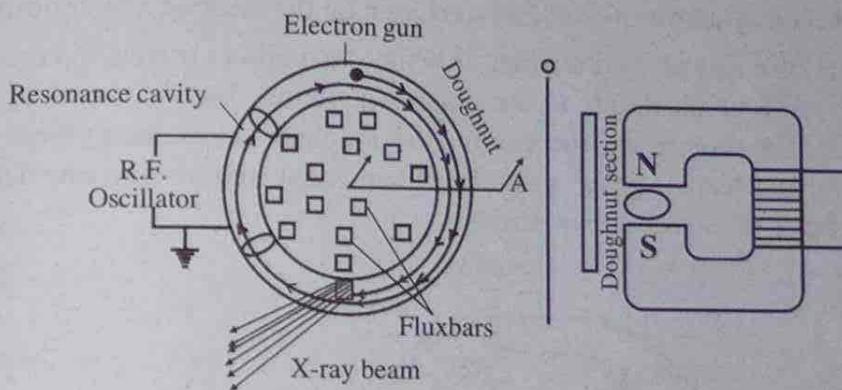
Figure 4.7

Working

Electrons are injected into the doughnut after a preliminary acceleration in an electrostatic field to 50 to 100 keV. When these are injected at the beginning of the magnetic cycle, the device operates as a betatron. The electrons travel in circular paths and increase their energy as the field increases until the electrons reach some 2MeV at which point the bars are magnetically saturated and are no longer able to induce an emf. This saturation causes the betatron mechanism to stop. At this point the synchrotron mechanism begins to operate. If the potential applied to the resonator operates at the proper frequency, the electrons are all kept in phase and receive increments of energy at each revolution, as they pass through the cavity. The high frequency field remains on while the magnetic field is increasing and is automatically cut off when the electrons have acquired their maximum energy. As the magnetic field still has a little increase to go before reaching its maximum,, the radius of

the beam of constant velocity ($\approx c$) diminishes a little so that the beam spirals in slightly to strike a target projecting from the inner edge of the doughnut which gives off short wavelength x-rays. The rays emerge in pulses as in the betatron. The maximum energy of the electrons depends on the radius and on the maximum magnetic field. The electron energies obtained is of the order of 1 GeV, although a 6 GeV electron synchrotron is now operating in the Soviet Union.

Proton synchrotron

Proton synchrotrons are used to produce energies of the order of several giga electron volts ($\sim 10\,\text{GeV}$). This much energy is necessary to probe into the nucleus. If this is done with synchrocyclotron the size and cost would be enormous. To overcome this proton synchrotron was derived based on the electron synchrotron.

It consists of a ring shaped magnet in which the particle travels in constant radius. There are four quadrants to the magnet covering the annular doughnut as shown figure. The protons are injected into the doughnut at low energy from a linear accelerator or by a Van de graaff machine and are recovered by magnetic deflection as a pulsed beam after many revolutions.
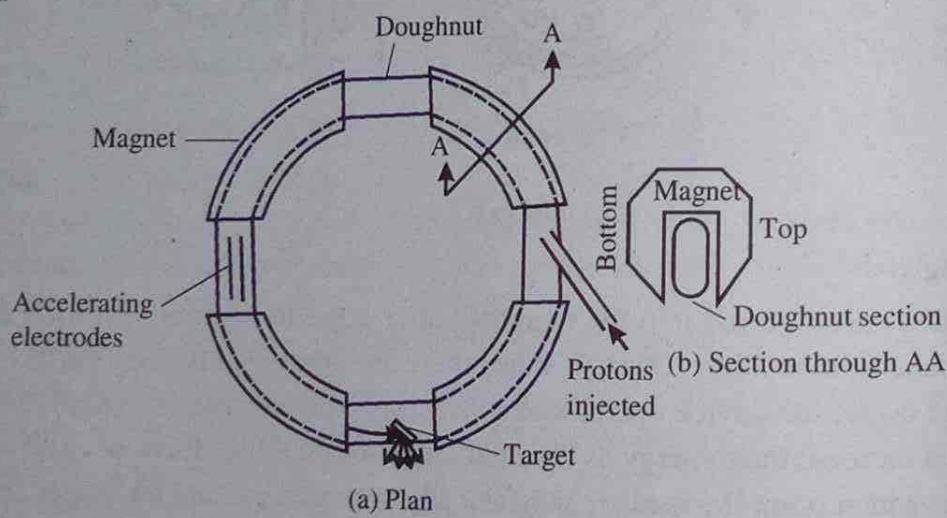
Figure 4.8: Proton Synchrotron

A high frequency resonator cavity accelerator is used in one of the straight parts with an increasing frequency corresponding to the increased speed of the protons. The field strength of the magnets is also increased to maintain the accelerated protons in a circular path of constant radius. The synchrotron action is applied at the beginning of each cycle. Energies attained are at the order of 10 GeV. The biggest present day machines are the Bevatron in the U.S.A which gives 6.4 GeV protons and the synchrophasotron of the U.S.S.R which operates at 10 GeV.

The alternating gradient synchrotron

It is a super proton synchrotron. We found that proton synchrotron is a constant gradient synchrotron, which yields an energy of about 10 GeV. If we want to have a proton energy of 50 GeV, such a machine would require a mass of 10^5 tonnes. It is found that the proton beam deviates appreciably from the circular path when radial magnetic field is constant. Since the whole doughnut is enclosed by the magnetic field in order to confine the beam as much as possible to a circular orbit, the size of the magnet becomes an important factor in the design of the big proton synchrotrons. For instance the equivalent mass of 50 GeV proton is 50 GeV + rest mass which is equal to 50 GeV + 0.938 GeV = 50.938 GeV.

Converting into atomic mass unit

$$m = 54.68\,u$$

or $$m = 54.68 \times 1.66 \times 10^{-27}\,kg$$

$$m = 54.68 \times 1.66 \times 10^{-27}\,kg$$

Using $$R = \frac{mv}{qB} = \frac{90.78 \times 1.66 \times 10^{-27} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 2}$$

Take B = 2 tesla

$$R = 85m$$

The orbital radius 85m and a magnetic field 2T is required.

To overcome this difficulty the alternating gradient synchrotron is devised. A method of overcoming this difficulty of beam wandering is to use magnetic fields with alternating gradients to focus the beam, i.e. in successive sections of the radial field, the gradient is first towards the centre and then outwards from the centre and soon. As the beam travels around its orbit it then passes through sections of the pole pieces of the magnet which cause the beam to be focused vertically and horizontally in rapid succession. At the same time the particles are defocused horizontally and vertically in rapid succession. All this is done by suitably arranging the magnetic field gradients to be inward and outward in the successive sections as described. Thus the field can be regarded as having an alternating gradient and this makes for the strong focusing and defocusing action. The A.G proton synchrotron in the Brook haven producing 33 GeV energy, which has more than 200 sections, each section is separated by a field free region and the total mass of the magnet is about 4000 tonnes the same as that of the C.G. proton synchrotron at Brookhaven. The A.G. Proton synchrotron at CERN (European Council for Nuclear research) at Geneva gives 25 GeV protons. A 70 GeV A.G. proton synchrotron has been operating at Serpukhov in the U.S.S.R.

After the ISR shut down in 1984 CERN's focus shifted to the large electron-positron collider (LEP) but ISR tunnel is still used for storage and magnetic purposes. The proton -antiproton collider was commissioned in the year 1981. After 10 years of functioning it was shutdown in the year 1991. After that CERN constructed still higher energy particle accelerator called relativistic heavy ion collider. it started operation in the year 2000 and still continuing. In the year 2009, large hadron collider (LHC) started its operation. We can see the details of LHC in the next section.

Intersecting beam accelerators

Physicist at CERN had planned to construct an intersecting beam accelerator in 1960s, based on the idea that smashing two particles beams head on would give higher energies than a single beam of particles with a fixed target. For this they constructed a specially designed device called intersecting storage ring (ISR). ISR composed of two interlaced rings each with a diameter 300 metres. Each ring contained a beam pipe surrounded by magnets to direct the circulating particles inside. Two protons can collide in the intersecting storage rings. The ISR begun its operation on 27th January 1971. It was the world's first hadron collider. ISR continued its operation until 1984.

Working of intersecting beam accelerator

Protons from a small proton linear accelerator (LINAC) A enter a booster accelerator. After boosting, protons enter into a proton synchrotron accelerator, where it is energised to about 30 GeV.

The energised protons, emerging from the proton synchrotron, are split magnetically at C. One beam goes through the ISR pipe in clockwise direction the other beam goes through the second ISR pipe in anticlockwise direction. The two beams collide at the eight intersections of two pipes of ISR. The schematic diagram of beam accelerator is given below.
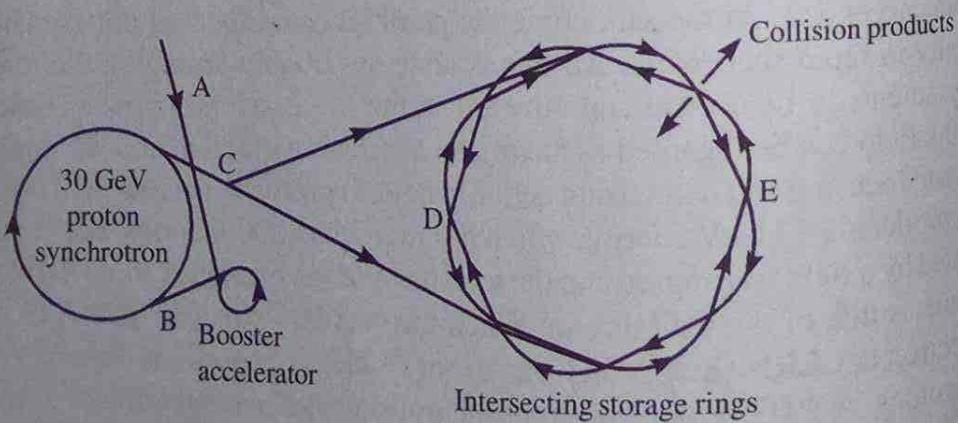

Figure 4.9: Intersecting beam accelerator

The maximum energy attained in ISR was 67GeV. This is equivalent to 2000 GeV beam hitting a stationary target. ISR paved the way for later accelerators as it hinted at the smaller elements of protons are now known to be quarks.

The growth and future of large accelerating machines

It has been realised that the mysteries of the ultimate structure of protons, neutrons and pions can only be solved by using intense beams of nuclear particles of very high energies as probes. The Cockcroft-Walton proton accelerator operated in 1932 the energy obtained was about 1MeV. The limiting value of this accelerator was 1 MeV. This is because at 1MeV the resistance of the accelerating tube materials break down. This led to the design of Van de graaff generator produced energy about 10 MeV.

In order to go beyond 10 MeV an entirely new method was required. The idea of resonance acceleration was conceived by Lawrence and living stone and mud the first cyclotron accelerator in 1932 which produced a proton beam about 1.2 MeV. In 1940 the betatron was designed and produced a beam of 2.3 MeV electrons. The energy limit of this was 25 MeV for protons and 300 MeV for electrons but linear accelerator was capable of producing 50 MeV for protons and about 1 GeV for electrons. The next break through was the construction of synchrotrons for both protons and electrons. The electron synchrotrons produced energies of the order of 350 MeV for electrons while the proton synchrotron produced energies upto 10 GeV. Finally the A.G. proton synchrotron was constructed and given output of 33 GeV, at the Brook Haven national laboratory. At present we arrived at the construction of 100 GeV energy output in the CERN (European Council for Nuclear research) at Geneva, Switzerland. So many European countries are contributing to this project. India had committed to contribute in kind 25 million U.S. dollars to L.H.C (Large Hadron Collider Project). CAT (Centre for Advanced Technology, Indore) is co-ordinating this activity.

In India major accelerator programmes are being persuaded at BARC (Baba Atomic Research Centre) TIFR (Tata Institute of Fundamental Research) at Mumbai, VECC (Variable Energy Cyclotron at Calcutta), NSC (Nuclear science centre) at Delhi.

CERN and its future project

The biggest accelerator of the world is at Geneva, Switzerland named CERN (European Council for Nuclear Research). The accelerator complex at CERN is a succession of machines that accelerates particles into increasingly high energies. Each machine booster the energy beam of particles before injecting the beam into the next machine in the sequence. The last element in the chain of machines is the large hadron collider (L.H.C). In the L.H.C beams are accelerated upto 6.5 TeV per

beam. Most of other accelerators (totally eight) in the chain have their own experimental halls, where beams are used for experiments at lower energies.

Working

Basically it is a proton accelerator. The proton is a source sample bottle of hydrogen. An electric field is used to strip of electrons from the hydrogen atoms to yield protons. Linac-2 the first accelerator in the chain accelerates the protons to the energy 50 MeV. This beams is injected into the next proton synchrotron (PS) accelerator booster which accelerates the protons to 1.4 GeV. This will be followed by the another proton synchrotron which pushes the beam to 25 MeV. Protons are then sent to super proton synchrotron (SPS) where they are accelerated to 450 GeV. These protons are finally transferred to the two beam pipes of the LHC. The beam in one pipe circulate clockwise while the other one anticlockwise over a length of 27 kilometres. The LHC ring is installed underground at a depth of 100 metres crossing the borders between Switzerland and France. Proton beam takes 4 minutes and 20 seconds to fill each LHC ring, and in 20 minutes protons to reach their maximum kinetic energy 6.5 TeV. The beams circulated for many hours inside the LHC beam pipes under normal operating conditions. Two beams are brought into collision inside four detectors ALICE(a large ion collider experiment) ATLAS (atmospheric laboratory for applications and science) CMS (compact muon solenoid) and LHCb (large hadron collider b) where total energy at collision point is equal to 13 TeV.

The CERN is fully operational now. There are totally 17,500 scientists, engineers and technicians are working over there.

The future project of CERN is to construct an accelerator 4 times larger and 6 times powerful than the current LHC. The circle collider will be a total length of 100 kilometres and cross over Geneva and other two countries. The installation cost would be about 21 billion Euros and expected to be operational in the year 2040.

IMPORTANT FORMULAE

1. In a Cockcroft-Walton proton accelerator the output voltage (V_0) of the voltage multiplier

$$V_0 = 2NV_p$$

Where N is the number of stages and V_p is the peak input voltage.

2. In a linear accelerator, velocities are in the ratio

$$V_1 : V_2 : V_3 \dots\dots\dots = 1 : \sqrt{2} : \sqrt{3} : \dots\dots$$

The lengths of the tubes must also be in the same ratio.

3. Cyclotron:

    a) Principle, $\dfrac{mv^2}{r} = Bqv$

b) Maximum kinetic energy of the particle, $K.E_{max} = \dfrac{B^2 q^2 r_0^2}{2m}$

c) Cyclotron frequency, $\upsilon = \dfrac{qB}{2m}$

d) Radius of the circular orbit $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2meV}}{qB}$

4. Betatron:

a) Betatron condition, $\dfrac{d\phi}{dt} = 2\pi r^2 \dfrac{dB}{dt}$

b) Average energy per turn, $\langle E \rangle = \dfrac{2e\phi_0 \omega}{\pi}$

c) Number of revolutions described by the electron, $N = \dfrac{c}{4\omega r}$

d) Final energy of the electron $= \dfrac{ec\phi_0}{2\pi r} = ec\,Br$

UNIVERSITY MODEL QUESTIONS

Section A

*(Answer questions in **two** or **three** sentences)*

Short answer type questions

1. What are accelerators? What is their uses?
2. What is the principle of Cockcroft-Walton accelerator?
3. What is a voltage doubler?
4. Write down the reaction of the first artificially transformed element using Cockcroft-Walton accelerator?
5. What is a linear accelerator?
6. What are the limitations of linear accelerators?
7. What is the principle of linear accelerator?
8. What are the uses of linear accelerators?
9. What is the difference between a linear accelerator and a cyclotron?
10. What is the principle of cyclotron?
11. What is cyclotron frequency?
12. Write down an expression for cyclotron frequency and explain the symbols used?
13. Can electrons be accelerated by cyclotron. Why?
14. What is the difference between electrostatic accelerators and cyclic accelerators?
15. What is a synchrotron?
16. What is a betatron?
17. What is the basic difference between a cyclotron and a betatron?
18. Write down the betatron condition and explain the symbols used?

19. What is an electron synchrotron?
20. What is a proton synchrotron?
21. What is the use of proton synchrotron?
22. What is the difference between electron and proton synchrotron?
23. What is the disadvantage of proton synchrotron?
24. What is an intersecting beam accelerator?
25. What is the basic idea of intersecting beam accelerator?
26. Draw a diagram of intersecting beam accelerator and label it?
27. What is an ISR?
28. Give the names of three accelerators still in use:

Section B

*(Answer questions in a **paragraph** of about half a page to one page)*

Paragraph / Problem type questions

1. Explain the working of a voltage doubler?
2. Explain the principle of working of linear accelerator?
3. What are the advantages of linear accelerators?
4. What are the limitations of a cyclotron ?
5. A charged particle moving in a uniform magnetic field penetrates a layer of lead and thereby loses half of its kinetic energy. How does the radius of curvature of its path change?
6. A charged particle of mass m and charge q revolves with a velocity v in a magnetic field B. Show that the radius r is proportional to its momentum?
7. How do the relativistic effects limit the acceleration of electrons to high energies in cyclotron?
8. What is the principle of synchrotron?
9. Explain two ways in which relativistic effect in accelerators can be overcome.
10. Explain the principle of betatron?
11. What are the uses of betatron?
12. What is the principle of electron synchrotron?
13. Explain the principle of alternating gradient synchrotron?
14. What is the principle of intersecting beam accelerator?
15. In a cyclotron the strength of the magnetic field is 0.954 Wb m^{-2} and the dees are dimensioned so that the accelerated protons emerge tangentially at a distance of 0.15m from the centre. What should be the frequency of the oscillator and the energy in MeV of the emerging protons. $m_p = 1.67 \times 10^{-27}$ kg   [14.55MHz, 0.98 MeV]
16. A cyclotron is to be designed to accelerate α-particles to a maximum energy of 32MeV. If the peak value of the accelerating voltage is 40 KeV, how many revolutions will be needed for a particle initially at rest before it acquires this amount of energy. If the induction associated with the magnetic field is 60 Wb m^{-2}. Compute the value of the diameter of the dees and frequency of the oscillator.

$$[n = 200 \quad d = 1.18m \quad \upsilon = 12.2MHz]$$

17. A cyclotron used to produce a beam of protons has a magnetic field of flux density $1.80 \, Wb \, m^{-2}$ and the extreme useful radius is 0.2m. Assume the specific charge of the proton $9.58 \times 10^7 \, C \, kgm^{-1}$, obtain the values of a) energy of the protons b) the wavelength of the electron magnetic waves produced by the oscillator. $m_p = 1.67 \times 10^{-27} \, kg$.

$$\begin{bmatrix} a) \; E = 6.2 \; MeV \\ b) \; \lambda = 10.92m \end{bmatrix}$$

18. An α-particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion if it enters normally in a region of thickness 0.1m having transverse magnetic induction of 0.1 J. Mass of α-particle $= 6.4 \times 10^{-27}$. $\qquad\qquad$ [30°]

19. Prove that the radius of curvature r of the path of a positive ion in a cyclotron is proportional to $\sqrt{n}$, where n is the number of times the ion, initially at rest has been accelerated across the space separating the two dees, provided the mass of the ion remains constant.

20. In a betatron the maximum magnetic field at orbit was $0.4 \, Wbm^{-2}$ operating at 50 Hz with a stable orbit diameter of 1.524 m. Calculate the average energy gained per revolution and the final energy of the electrons. $\qquad$ [291.9eV, 91.45MeV]

21. A cyclotron with dees of diameter 1.8 m has a magnetic field of 0.8T. Calculate the energy of the accelerated protons $\qquad\qquad\qquad\qquad$ [25 MeV]

22. An electron and a proton have 1MeV energies each. Which one should be treated relativistically. $\qquad\qquad\qquad\qquad\qquad\qquad$ [only electron]

23. In a betatron accelerator the maximum magnetic field at orbit was 0.4T, operating at $50 \, H_z$ with a stable orbit diameter of 1.5m. Calculate the average energy gained per revolution and the final energy of the electrons. $\qquad$ [239.5eV, 91 MeV]

24. A cyclotron with dees of radius 0.9m has a magnetic field of 0.8T. Calculate the generator frequency required to accelerate the proton to 25 MeV. $\qquad$ [6.17 × 10⁷ Hz]

25. What radius is needed in proton synchrotron to attain particle energies of 10 GeV. Assume that magnetic field available is 1.5T $\qquad\qquad\qquad\qquad$ [18m]

Section C

*(Answer questions in about **one** or **two** pages)*

Long answer type questions (Essays)

1. Explain the principle, construction and working of a Cockcroft-Walton accelerator.
2. Describe the working of Van de graaff generator with a neat diagram.
3. Explain the principle, construction and working of a linear accelerator?
4. Explain with a neat figure the working of a cyclotron?
5. Describe the principle, construction and working of a betatron?
6. Describe the principle, construction and working of an electron synchrotron?
7. Explain a proton synchrotron in detail?
8. Explain an alternating proton synchrotron in detail?
9. Write down an essay on the growth and future of accelerating machines?

10. Explain the principle, construction and working of an intersecting beam accelerator with a diagram?

Hints to problems

15. $\upsilon = \dfrac{qB}{2\pi m}$, $E = \dfrac{B^2 q^2 r_0^2}{2m}$, $r_0 = 0.15m$

16. a) The particle will be accelerated twice during each revolution. For one revolution energy acquired $= q.2V$. Let n be the required number of revolution to get 32 MeV

$\therefore$ $n\, q.2V = 32\,MeV$

$q = 3.2 \times 10^{-19}C$, $V = 40 \times 10^3 V$

b) $\dfrac{B^2 q^2 r_0^2}{2m} = 32 \times 10^6 \times 1.6 \times 10^{-19}$ find r_0, $\because d = 2r_0$

c) $\upsilon = \dfrac{qB}{2\pi m}$
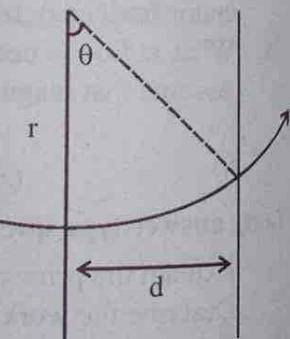
17. a) $E = \dfrac{B^2 q^2 r_0^2}{2m} = \dfrac{B^2 r_0^2}{2}\left(\dfrac{q}{m}\right)^2 m$

b) $\lambda = \dfrac{c}{\upsilon}$, $\upsilon = \dfrac{qB}{2\pi m}$

18. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2\,m\,k\cdot E}}{qB} = \dfrac{\sqrt{2m\,qV}}{qB}$ find r $r = 0.2m$

$\sin\theta = \dfrac{d}{r}$

$\theta = \sin^{-1}\dfrac{d}{r} = \sin^{-1}\left(\dfrac{0.1}{0.2}\right)$

19. We have $\dfrac{mv^2}{r} = Bev$ or $mv = Ber$. If V be the maximum value of the p.d between the dees $\dfrac{1}{2}mv^2 = eVn$

or $m^2 v^2 = \dfrac{2eVn}{m}$ or $mv = \sqrt{\dfrac{2eVn}{m}}$

using $\quad r = \dfrac{mv}{Be} = \sqrt{\dfrac{2eVn}{m}} \cdot \dfrac{1}{Be}$

$\qquad r = \dfrac{1}{B}\sqrt{\dfrac{2V}{me}} \cdot \sqrt{n}$

or $\qquad r \propto \sqrt{n}$

20. See example 6. $N = 3.132 \times 10^5$

21. $KE_{max} = \dfrac{B^2 q^2 r_0^2}{2m} = \dfrac{0.8^2 \times (1.6 \times 10^{-19})^2 \times (0.9)^2}{2 \times 1.67 \times 10^{-27}} J$

$\qquad = \dfrac{0.8^2 \times (1.6 \times 10^{-19})^2 \times 0.9^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-13}} MeV$

$\qquad = 24.83\ MeV \simeq 25\ MeV$

22. $K.E = (m - m_0)c^2 = m_0 c^2 \left(\dfrac{m}{m_0} - 1\right)$

$\qquad K.E = E_0 \left(\dfrac{m}{m_0} - 1\right), \quad E_0 = m_0 c^2 = Rest\ energy$

$\therefore \qquad \dfrac{m}{m_0} = 1 + \dfrac{K.E}{E_0}$

For a proton

$$\left(\dfrac{m}{m_0}\right)_{proton} = 1 + \dfrac{1MeV}{938MeV} = 1.00106$$

So proton with energy need not be considered relativistically.

For an electron

$$\left(\dfrac{m}{m_0}\right)_{electron} = 1 + \dfrac{1MeV}{0.51MeV} = 2.96$$

So electron with energy 1 MeV should be considered relativistically.

23. Here the electron must be treated relativistically

$$mv = \dfrac{E}{c}, \quad E\ is\ the\ final\ energy.$$

Using $\dfrac{mv^2}{R} = Bqr$

$\therefore \qquad mv = BqR$

$\dfrac{E}{c} = BqR$

$\therefore \qquad E = BqR\,c$

Substituting the values, we get $E = 91$ MeV.

Total number of revolutions

$$N = \dfrac{c}{4\omega R} = \dfrac{3 \times 10^8}{4 \times 2\pi \times 50 \times 0.75} = 3.1 \times 10^5$$

$\therefore$ Average energy per revolution

$$= \dfrac{91 \times 10^6 \,\text{eV}}{3.1 \times 10^5} = 293.5 \text{ eV}$$

24. $\upsilon = \dfrac{qB}{2m}$ ..... (1)

$$K.E = \dfrac{B^2 q^2 r_0^2}{2m} \qquad\qquad ..... (2)$$

$\dfrac{Eq(1)}{Eq(2)}$ gives $\dfrac{\upsilon}{K.E} = \dfrac{1}{Bqr_0^2}, \quad \upsilon = \dfrac{K.E}{Bqr_0^2}$

$$\upsilon = \dfrac{25 \times 10^6 \times 1.6 \times 10^{-19}}{0.8 \times 1.6 \times 10^{-19} \times (0.9)^2} = 6.17 \times 10^7 \,\text{Hz}$$

25. $R = \dfrac{mv}{Bq}$, Equivalent mass of 10 GeV proton is equal to 10 GeV + rest mass.

$m = 10.938$ GeV

$m = 11.75$ u

$m = 11.75 \times 1.66 \times 10^{-27}$ kg

$m = 1.95 \times 10^{-26}$ kg

Take $v = 3 \times 10^8 \,\text{ms}^{-1}$

⑤ ELEMENTARY PARTICLES

Introduction

For many decades physicists have been searching for the ultimate building blocks of particles of which all matter is composed of. These building blocks or particles are elementary particles. After studying the structure of atoms we believed that the electrons, protons and neutrons are the only building blocks of matter (elementary particles). As we look more deeply, although the nature has constructed all material objects out of roughly 100 kinds of atoms, we can understand these atoms in terms of electrons, protons and neutrons. Our attempts to look further within the electron, it seems to be a fundamental particle in the sense that it has no internal structure. i.e. they cannot be explained by as a system of other particles. With the advent of high energy particle accelerators a large number (above 200) of elementary particles have been discovered. If all of these are fundamental particles; it would be impossible to formulate any fundamental dynamic laws of their behaviour. However experiments show that there are only few fundamental building blocks of nature. All others could be explained in terms these fundamental particles. Apart from electron, quarks are fundamental particles. In this chapter we discuss the properties of many of the particles discovered so far and laws that govern their behaviour. Finally we classify these particles. We also discuss about quarks and their properties.

The four basic forces

Physics people are familiar with several forces such as frictional force, tension, viscous force, surface tension, gravitational, electric, magnetic, nuclear radiation, radioactive.... etc. All of these known forces can be grouped into four basic types. In order of increasing strength, these are gravitation, electromagnetism, weak interaction and strong interaction.

1. Gravitational interaction

It is the force between any two masses govern by the equation $F = \dfrac{GMm}{r^2}$. It is the weakest force in nature and its range is infinity. On the microscopic scale it is

very important, but on the microscopic scale it is of no importance at all. However, though the tiny gravitational interaction between one atom of the Earth and one atom our body is negligible, the combined effect, the interactions between all atoms of Earth and all atoms of our body, is observable. The strength of the gravitational force comparing to strong force is 10^{-38}. Every interaction is mediated by a particle. The particle responsible for this interaction is called graviton. It is a massless particle with spin 2. It is due to its weakest strength, it has not been discovered yet.

2. The weak interaction

The forces involved in the process of beta decay and other similar processes producing particles. The weak force between two neighbouring protons is about 10^{-7} times of the strong force between them. This shows that the weak force does not play any role in the binding of nuclei (strong force). The range of this force is very short of the order of 0.001 fm. Nevertheless, the weak force is important in understanding the behaviour of fundamental particles and is critical in understanding the evolution of the universe. The particle responsible for weak interactions are called vector bosons. There are two types of vector bosons. They are (i) $W^{\pm}-$ boson and (ii) $Z^{0}-$ boson. Since the range of the weak interaction is small, vector bosons are massive. The W – vector boson has a spin 1 and charge $\pm$ e and is responsible for ordinary beta decays. Its mass is about 85 times the proton mass (80.4 GeV) But Z^{0}– vector boson is chargeless and has spin 1 and its mass is about 97 times the proton mass (91.2 GeV).

3. The electromagnetic interaction

It is the force due to interaction between fundamental particles. The macroscopic forces such as friction, air resistance, viscous, tension etc. are electromagnetic forces at the atomic level. Within the atom electromagnetic force dominate. The electromagnetic force between neighbouring protons in a nucleus is about 10^{-2} times the strong force between them. Within the nucleus the electromagnetic forces can affect cumulatively hence cannot be neglected with reference to strong force. In determining the stability and structure of nuclei, these two forces have to be considered.

The range of this force is infinity and the particle responsible for this force is photon. Photon is a massless particle with spin1.

4. The strong interaction

This is the force responsible for the binding of nuclear or of that between quarks. In nuclear reactions and decays of fundamental particles this force dominates. Some

particles (such as electron) do not feel this force at all. The range of this force is relatively very small of the order of 1fm.

The relative strength of a force determine the time scale over which it acts. Its characteristic time scale is less than 10^{-22} s, whereas in the case of electromagnetic force its characteristic time scale over which it acts ranges from 10^{-14} s to 10^{-20} s. For weak force it is $10^{-8} - 10^{-13}$ s.

The particle responsible for strong nuclear force is pion, where as gluon is the particle responsible for the strong force between quarks.

Table 5.1: Four basic forces and their properties
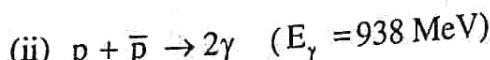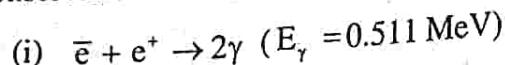
Type	Range	Relative strength	Characteristic time (s)	Field particle	Charge	Spin ($\hbar$)	Rest Energy (GeV)
Strong	1 fm	1	$< 10^{-22}$	Pion or Gluon	+e, −e, 0 or 0	1 1	0
Electro Magnetic	∞	10^{-2}	$10^{-14} - 10^{-20}$	Photon	0	1	0
Weak	10^{-3} fm	10^{-7}	$10^{-18} - 10^{-13}$	Vector bosons W⁺, W⁻ and Z⁰	+1, −1 and 0	1 1	80.4 91.2
Gravita tional	∞	10^{-38}	years	Graviton	0	2	0

Particles and antiparticles

Protons, neutrons and electrons are particles of which ordinary matter is made. Experiments shown that every particle has an antiparticle. Antiparticles have the same mass and spin as the particles but opposite charge and magnetic moment. For example positron (e^+) is the antiparticle of electron ($\bar{e}$) antiproton ($\bar{p}$) is the antiparticle of proton (p). Four particles the photon, the graviton, neutral pion (π^0), neutral eta meson (η^0) are their own antiparticles. Matter is made up of particles where as antimatter is made up of antiparticles (antiproton, antineutron and positrons)

For example a stable atom of antihydrogen could be constructed from a positron and an antiproton. The properties of this atom would identical to those of ordinary hydrogen atom. Antiparticles of stable particles (such as the positron and the anti

proton) are themselves stable. However, when a particle and its antiparticle meet the annihilation reaction can occur. In this reaction particle and antiparticle annihilate to form two or more photons. This reactions occurs in accordance with law of conservation of momentum and energy. Examples :

(i) $\bar{e} + e^+ \rightarrow 2\gamma$ $(E_\gamma = 0.511 \text{ MeV})$

(ii) $p + \bar{p} \rightarrow 2\gamma$ $(E_\gamma = 938 \text{ MeV})$

Note: An antiparticle is designated by the same symbol as the particle but with a bar over it.

Neutrions are chargeless elementary particles. There are three types of neutrinos. They are electron neutrino (υ_e), muon neutrino (υ_μ) and tau neutrino (υ_τ). Though the electron neutrino and muon neutrino are massless tau neutrino has mass about $320 \, m_e$ (where m_e is the mass of the electron). These particles have corresponding antiparticles. They are antielectron neutrino ($\bar{\upsilon}_e$), antimuon neutrino ($\bar{\upsilon}_\mu$) and anti tau neutrino ($\bar{\upsilon}_\tau$).

Since ordinary matter is not composed of neutrino, we cannot distinguish between neutrino and anti neutrino on the basis of matter and antimatter. But the conservation laws in beta decay process can be understood most easily if we define the antineutrino to be the particle that accompanies β^- decay and neutrino accompanies β^+ decay. Another example is consider a lambda (Λ) particle decays to an ordinary neutron. Then antilambda ($\bar{\Lambda}$) decays into antineutron. Similarly negative muon (μ^-) and positive muon (μ^+) are antiparticles of one another because μ^- decays to e^- and μ^+ decays to e^+.

Families of particles

There are more than 200 elementary particles have been discovered. One way to study them is to classify it into different categories based on certain properties and then look for similarities among the classifications. We have already classified according to their types of forces through which their interactactions. In early days of particle physics they were classified according to their masses. It was observed that the lightest particles (electrons, muons and neutrinos) showed one type of behaviour, the heavy mass particles (protons and neutrons) showed a different behaviour and the medium mass particles (pions and kaons) showed a still different behaviour. The Greek word for light is lepton, that for medium is meson and for heavy it is baryon. So elementary particles were three types. Leptons, mesons and

baryons. Gradually the original meaning attached to the terms lost their meaning. For example the term lepton was used for light particles, but later heavy leptons were discovered so the name lepton became unsuitable. The classification of particles by mass is now obsolete but we retained the names leptons, mesons and baryons. Now these names describe a group or family of particles with similar properties.

Leptons

Leptons are weakly interacting fermions having spin $\frac{1}{2}\hbar$. These particles take part in weak interactions and electromagnetic interactions when charged. There are six members of this family. They are electron (e⁻), electron neutrion (υ_e), muon (μ^-), muon neutrino (υ_μ), tau (τ^-) and tau neutrino (υ_τ). Ofcourse this six family members of leptons have corresponding antiparticles together called antileptons. Positron (e⁺), antielectron neutrino ($\overline{\upsilon}_e$), antimuon (μ^+), antimuon neutrino ($\overline{\upsilon}_{\mu^+}$), antitau ($\tau^+$) and antitau neutrino $\overline{\upsilon}_\tau$ are antileptons. The symbol, charge and mass of lepton family are given in the table below.

Table 5.2
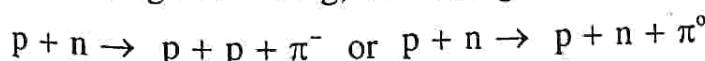
Name	Symbol	Charge	Mass in terms of electron m_e
1. Electron	e⁻	−1	1
2. Electron neutrino	υ_e	0	~0
3. Muon	μ^-	−1	206.7 m_e
4. Muon neutrino	υ_μ	0	~0
5. Tau	τ^-	−1	3500 m_e
6. Tau neutrino	υ_τ	0	<320 m_e

The lepton family

Note: Fermions are particles with half integral spin.

Mesons

Mesons are strongly interacting particles having integral spin. Mesons can be produced in reactions through the strong interactions; they decay to other mesons or leptons through the strong, electromagnetic or weak interactions. For example

$$p + n \rightarrow p + p + \pi^- \quad \text{or} \quad p + n \rightarrow p + n + \pi^\circ$$

This is strong interaction. The pions further decay according to $\pi^- \rightarrow \mu^- + \bar{\upsilon}_\mu$ with mean life time 2.6×10^{-8} s, so this is weak interaction.

π° decays according to $\pi^\circ \rightarrow \gamma + \gamma$ with mean life time 8.4×10^{-17} s. So this is electromagnetic interaction.

Since mesons are not observed in matter, the classification into particles and anti particles become purely arbitrary. For example π^+ and π^- are antiparticles of one another. Similarly K^+ and K^-. For uncharged mesons such as π° and η°, the particle and antiparticles are identical. The antiparticle of neutral kaon (K°) is ($\bar{K}^\circ$). K° and $\bar{K}^\circ$ are distinct. A list of mesons with their properties is given in table below.

Table 5.3: Some selected mesons
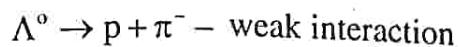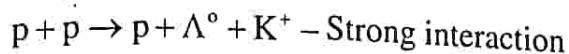
Particle	Anti particle	Charge of particle	Spin $\hbar$	Strangeness of particle	Rest energy (MeV)
π^+	π^-	+1	0	0	140
π°	π°	0	0	0	135
K^+	K^-	+1	0	+1	494
K°	$\bar{k}^\circ$	0	0	0 + 1	498
η	η	0	0	0	548
ρ^+	ρ^-	+1	1	0	775
η'	η'	0	0	0	958
O^+	O^-	+1	0	0	1869
J/ψ	J/ψ	0	1	0	3097
γ	γ	0	1	0	9460

Note: Strangeness of the particles will be discussed soon.

Baryons

The baryons are strongly interacting particles with half integral spins $\left(\dfrac{1}{2}, \dfrac{3}{2}, \dfrac{5}{2} \dots \right)$. Baryons have distinct antiparticles like leptons. Baryons are produced

in reactions through strong interactions and decay through electromagnetic and weak interactions. Example.

$$p + p \rightarrow p + \Lambda^\circ + K^+ - \text{Strong interaction}$$

$$\Lambda^\circ \rightarrow p + \pi^- - \text{weak interaction}$$

Some selected baryons and their properties are given in table 5.4

Table 5.4: Some selected baryons

Particle	Antiparticle	Charge of particle	Spin $\hbar$	Strangeness of particle	Rest energy (MeV)
p	$\bar{p}$	+1	$\frac{1}{2}$	0	938
n	$\bar{n}$	0	$\frac{1}{2}$	0	940
Λ°	$\bar{\Lambda}^\circ$	0	$\frac{1}{2}$	−1	1116
Σ^+	$\bar{\Sigma}^+$	+1	$\frac{1}{2}$	−1	1189
Σ°	$\bar{\Sigma}^\circ$	0	$\frac{1}{2}$	−1	1193
Σ^-	$\bar{\Sigma}^-$	−1	$\frac{1}{2}$	−1	1197
Ξ°	$\bar{\Xi}^\circ$	0	$\frac{1}{2}$	−2	1315
Ξ^-	$\bar{\Xi}^\circ$	−1	$\frac{1}{2}$	−2	1322
Δ^*	$\bar{\Delta}^*$	+2, +1, 0, −1	$\frac{3}{2}$	0	1232
Σ^*	$\bar{\Sigma}^*$	+1, 0−1	$\frac{3}{2}$	−1	1385
Ξ^*	$\bar{\Xi}^*$	−1, 0	$\frac{3}{2}$	−2	1533
Ω^-	$\bar{\Omega}^-$	−1	$\frac{3}{2}$	−3	1672

So far we were discussing the three families of particles. Their structure, interactions, spin and examples are given in Table 5.5 given below.

Table 5.5: Families of particles

| Family	Structure	Interactions	Spin	Examples
Leptons	Fundamental	weak, electro magnetic	Half integral	e, υ
Mesons	Composite	weak, electro magnetic, strong	Integral	π, K
Baryons	composite	weak, electro magnetic, strong	Half integral	p, n
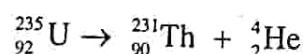
Conservation laws

Conservation laws form the back bone of physics. We are familiar with conservation laws like linear momentum, angular momentum, energy etc. These conservation laws are closely connected with the fundamental properties (symmetries) of space and time. We believe, not yet proved contrary, these laws are absolute and inviolable. In the decays and reactions of elementary particle, conservation laws provide a way to understand why some processes occur and others are not observed.

Apart from the well known conservation laws stated above, in nuclear physics there are several others like nucleon number conservation, charge conservation, lepton number conservation, baryon number conservation, strangeness conservation and somany others.

Nucleon number conservation

Consider the alpha decay of a nucleus, such

$$^{235}_{92}U \rightarrow \, ^{231}_{90}Th + \, ^{4}_{2}He$$

The proton number on the L.H.S is 92.
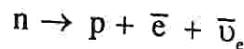
The proton number on the R.H.S. is $90 + 2 = 92$

The neutron number on the L.H.S $= 235 - 92 = 143$

The neutron number on the R.H.S. $= (231 - 90) + (4 - 2) = 143$

We balance the proton numbers and also neutron numbers on both sides. This does not mean that in nuclear reactions conserve both proton and neutron number. The next example will clarify this.

Consider the beta decay.

$$n \to p + \bar{e} + \bar{\upsilon}_e$$

This decay process does not conserve either neutron or proton number. However it conserves the total neutron number plus proton number before and after decay. This is called nucleon number conservation. In all the reaction decays this will be conserved.

Lepton number conservation
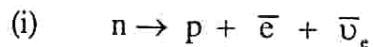
The lepton family consists of 6 members. Each member is assigned a number called lepton number (L). The corresponding antiparticles of lepton are assigned a number -1. If the particle is not a lepton, the lepton number is zero. i.e, for all mesons and baryons lepton number is zero. In all reactions and decays, the lepton number before after process must be conserved.

i.e., $L_{e^-} = 1, \quad L_{\upsilon_e} = 1, \quad L_{\mu^-} = 1, \quad L_{\upsilon_\mu} = 1, \quad L_{\tau^-} = 1, \quad L_{\upsilon_\tau} = 1$
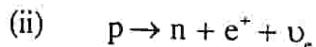
For antiparticles of leptons:

$$L_{e^+} = -1, \quad L_{\bar{\upsilon}_e} = -1, \quad L_{\bar{\mu}^-} = -1, \quad L_{\bar{\upsilon}_\mu^-} = -1, \quad L_{\bar{\tau}^-} = -1, \quad L_{\bar{\upsilon}_\tau} = -1$$

Examples

(i) $n \to p + \bar{e} + \bar{\upsilon}_e$

   L: $0 \to 0 + 1 + (-1)$

(ii) $p \to n + e^+ + \upsilon_e$
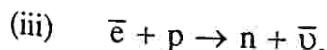
   L: $0 \to 0 + -1 + 1$
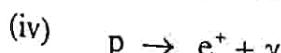
In both of the decays lepton number is conserved. The lepton number conservation accounts for the emission of antineutrino in β^- decay and the neutrino in β^+ decay.

(iii) $\bar{e} + p \to n + \bar{\upsilon}_e$

   L: $1 + 0 \to 0 + -1$

   $1 \to -1$

   since lepton number is not conserved this reaction is forbidden.

(iv) $p \to e^+ + \gamma$

   L: $0 \to -1 + 0$

The lepton number is not conserved, this decay is forbidden in nature.

After checking lepton number conservation in general, we have to check whether each type of lepton (e, μ, τ) number is the same on both sides. Then only conservation law is valid.

Examples

1. $p + \bar{\upsilon}_e \rightarrow e^+ + n$

     $L_e : 0 + -1 \rightarrow -1 + 0$

Thus electron type lepton number is conserved

2. $\upsilon_\mu + n \rightarrow \bar{\mu} + p$

     $L_\mu : 1 + 0 \rightarrow -1 + 0$

Here muon type lepton number is not conserved.

3. $\bar{\upsilon}_\mu + p \rightarrow n + e^+$

     $L: \quad -1 + 0 \rightarrow 0 + -1$

Here lepton number is conserved. But this is not observed. This is because muon type lepton number is not conserved.

     $L_\mu : -1 + 0 \rightarrow 0 + 0$

So muon type lepton number is not conserved.

Thus, lepton number conservation can be put in the following way:

In any process, the lepton numbers for electron type leptons, muon-type leptons and tau-type leptons must each remain constant.

Baryon number conservation

The baryon family also consists of 6 members (p, n, Λ°, Σ, Ξ°, Ω^-). Each member is assigned a number + 1 called baryon number (B). The corresponding antiparticles of baryon are assigned B = −1. If the particle is not a baryon the baryon number is zero. i.e, all mesons and leptons have B = 0.
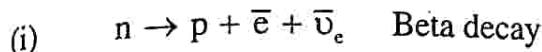
$$B_p = 1, \quad B_n = 1, \quad B_{\Lambda^\circ} = 1, \quad B_\Sigma = 1, \quad B_{\Xi^\circ} = 1, \quad B_{\Omega^-} = 1$$

For antiparticles
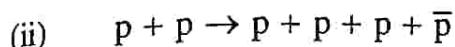
$$B_{\bar{p}} = -1, \quad B_{\bar{n}} = -1, \quad B_{\bar{\Lambda}^\circ} = -1, \quad B_{\bar{\Sigma}} = -1, \quad B_{\bar{\Xi}^\circ} = -1, \quad B_{\bar{\Omega}^-} = -1$$

Baryon number conservation states that in any process baryon number must be conserved before and after the process. It may be noted that the nucleon number conservation is a special case of baryon number conservation. Since nucleons are included in baryons we need to go for nucleon conservation separately.

Example

(i) $n \rightarrow p + \bar{e} + \bar{\upsilon}_e$ Beta decay
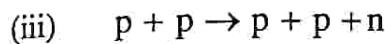
    B: $1 \rightarrow 1 + 0 + 0$ Baryon number conserved

(ii) $p + p \rightarrow p + p + p + \bar{p}$

    B: $1 + 1 \rightarrow 1 + 1 + 1 - 1$

    $2 \rightarrow 2$

Baryon number is conserved

(iii) $p + p \rightarrow p + p + n$
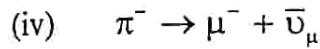
    B: $1 + 1 \rightarrow 1 + 1 + 1$

    $2 \rightarrow 3$

Here baryon number is not conserved, so this reaction is forbidden

(iv) $\pi^- \rightarrow \mu^- + \bar{\upsilon}_\mu$

    B: $0 \rightarrow 0 + 0$

Baryon number is conserved

Strangeness conservation

First in cosmic rays and later particles were discovered whose behaviour were rather peculiar. These particles were produced in strong interactions in time and short as 10^{-23} s but while decaying they took an enormously long time $\sim 10^{-8}$ s, characteristic of weak interactions. Such particles were called strange particles.

The strange behaviour of the particles was pointed by A Pais. The other strange property of these particles was that they produced in pairs. Why this strange behaviour? The answer to this question came from Gell Mann and Nishijima who assigned a new number to these particles called strangeness number(S). This number was defined analogous to charge specifically for a conservation law based on experimental data.

The strangeness numbers assigned to various particles are given below.

(i) For the kaons (K^+, K^0), $S = +1$

For the antiparticles $(\bar{K}^+, \bar{K}^0)$, $S = -1$
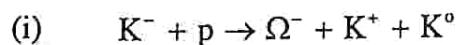
(ii) For the lambda hyperon (Λ^0), $S = -1$ and for the corresponding antiparticle $(\bar{\Lambda}^0)$, $S = +1$

(iii) For the sigma hyperons $(\Sigma^+, \Sigma^-, \Sigma^0)$, $S = -1$ and for the corresponding antiparticles $(\bar{\Sigma}^+, \bar{\Sigma}^-, \bar{\Sigma}^0)$, $S = +1$

(iv) For the cascade hyperons (Ξ^0, Ξ^-), $S = -2$ and for the corresponding antiparticles $(\bar{\Xi}^0, \bar{\Xi}^-)$, $S = +2$

(v) For the omega hyperon $((\Omega^-)$, $S = -3$ and for the corresponding antiparticle $(\bar{\Omega}^-)$, $S = +3$.

Examples

(i) $K^- + p \rightarrow \Omega^- + K^+ + K^0$
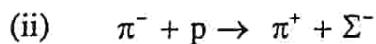
S: $-1 + 0 \rightarrow -3 + 1 + 1$

$-1 = -1$
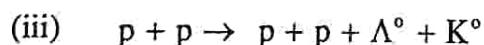
In this reaction strangeness number is conserved.

(ii) $\pi^- + p \rightarrow \pi^+ + \Sigma^-$

S: $0 + 0 \rightarrow 0 - 1$

$0 \rightarrow -1$

In this process strangeness number is not conserved. So this process is not allowed in nature.

(iii) $p + p \rightarrow p + p + \Lambda^0 + K^0$

S: $0 + 0 \rightarrow 0 + 0 - 1 + 1$

$0 \rightarrow 0$

So strangeness number is conserved. But we cannot say whether this process occurs or not. For this we have to check all other conservation laws. For example suppose we check baryon number conservation, it gives

$$B : 1 + 1 \rightarrow 1 + 1 + 1 + 0$$

$$2 \rightarrow 3$$

So baryon number is not conserved. Thus this process is not allowed.

A very important thing to be noted is that strong interactions and electromagnetic interactions can conserve strangeness number, but weak interaction decays cannot conserve strangeness number. Here strangeness changes by unity so we can write

$\Delta S = 0$ for strong interactions and electromagnetic interactions

$\Delta S = 0, \pm 1$ for weak interactions

Examples

(i) $\quad \Omega^- \rightarrow \Sigma^- + \pi^0$

$\quad$ S: $-3 \rightarrow -1 + 0$

$\therefore \quad \Delta S = -1 - -3 = 2$, so condition that for weak interaction $\Delta S = 0, \pm 1$ is violated

(ii) $\quad \Xi^0 \rightarrow n + \pi^0$

$\quad$ S: $-2 \rightarrow 0 + 0$

$\therefore \quad \Delta S = 2$

This being a weak interaction ΔS must be $0, \pm 1$, so strangeness is not conserved. Finally we can define the strangeness number conservation as follows.

In processes governed by strong or electromagnetic interactions the total strangeness number before and after the process must be conserved and in processes governed by the weak interaction strangeness number either remains constant or changes by one unit.

Note: In all processes in addition to above three conservation laws, charge must be conserved as well as spin also must be conserved.

Example 1

Why does a free neutron not decay into an electron and a positron.

Solution

Suppose a free neutron decays into an electron and a positron.

i.e., $\quad n \rightarrow e^- + e^+$

Bariyon quantum number

$$1 = 0 + 0$$

Since baryon quantum number is not conserved the decay is not possible.

Spin is also not conserved.

Example 2

Which of the following reactions can occur state the conservation principles violated by the others.

a) $\Lambda^\circ \rightarrow \pi^+ + \pi^-$

b) $\pi^- + p \rightarrow n + \pi^\circ$

c) $\pi^+ + p \rightarrow \pi^+ + p + \pi^- + \pi^\circ$

d) $\gamma + n \rightarrow \pi^- + p$

Solution

a) $\Lambda^\circ \rightarrow \pi^+ + \pi^-$

Applying baryon number conservation we get

B : $1 = 0 + 0$ since B is not conserved, this reaction cannot occur

b) $\pi^- + p \rightarrow n + \pi^\circ$

B : $0 + 1 = 1 + 0$ i.e. baryon number is conserved

Applying charge conservation we get

C : $-1 + +1 = 0 + 0$

i.e. charge is also conserved

Hence this reaction can occur.

c) $\pi^+ + p \rightarrow \pi^+ + p + \pi^- + \pi^\circ$

Applying charge conservation

C : $1 + 1 = 1 + 1 - 1 + 0$

i.e. charge is not conserved. Hence this reaction cannot occur.

d) $\gamma + n \rightarrow \pi^- + p$

B : $0 + 1 = 0 + 1$ Baryon number is conserved

C : $0 + 0 = -1 + 1$ charge is conserved.

Hence this reaction can occur.

Example 3

A muon (μ^-) collides with a proton, a neutron plus another particle is formed. What is the other particle.

Solution

$\mu^- + p \rightarrow n + x$, where x is the new particle formed.
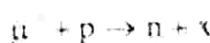
Applying charge conservation

$C: \quad -1 + 1 = 0 + x$

i.e. $\quad 0 = 0 + x$

This implies that x is chargeless.

Applying lepton number, we get

$$\mu^- + p \rightarrow n + x$$

$L_\mu: \qquad 1 + 0 \rightarrow 0 + x$

This implies that x is a μ-lepton. i.e. it is a mu-neutrino.

Example 4

A positive pion collides with a proton, two protons plus another particle are created. What is the other particle.

Solution

$\pi^+ + p \rightarrow p + p + x$, where x is the new particle formed.

Applying charged conservation, we get

$C: \quad 1 + 1 = 1 + 1 + x$

This implies that x is chargeless

Applying baryon number conservation, we get

$B: \quad 0 + 1 = 1 + 1 + x$

This shows that the baryon number of x is -1. i.e. it is an antiparticle of baryon. In other words x is a chargeless antibaryon.

Example 5

The products of a collision between a fast proton and a neutron are a neutron, a Σ^+ particle and another particle. What is the other particle.

Solution

$p + n \rightarrow n + \Sigma^\circ + x$, where x is the unknown particle formed.

Applying charge conservation, we get

C : $1 + 0 = 0 + 0 + x$

This implies that x is a charged particle (+ve)

Applying baryon number conservation

B : $1 + 1 = 1 + 1 + x$

This implies that baryon number of x is 0. i.e. x is not a baryon.

Applying spin conservation, we get

S : $\dfrac{1}{2} + \dfrac{1}{2} = \dfrac{1}{2} + \dfrac{1}{2} + x$

This shows that x is a spinless particle.

Applying strangeness conservation, we get

S : $0 + 0 = 0 - 1 + x$

This implies that the strangeness of x is 1. Hence x is a positively charged kaon.

Example 6

In proton-proton collision a lambda hyperon, a proton, a positively charged pion and a new a particle are formed. What is the new particle

Solution

$p + p \rightarrow \Lambda^\circ + p + \pi^+ + x$, where x is the new particle formed

Applying charge conservation, we have

C : $1 + 1 = 0 + 1 + 1 + x$

This implies that the new particle formed is chargeless.

Since Λ° is a strange particle and the strange particles are formed in pairs. Thus x must be a strange particle.

Applying strangeness quantum number conservation, we have.

S : $0 + 0 = -1 + 0 + 0 + x$

This implies that x is a particle with strangeness +1.

Hence x is a chargeless kaon (K°).

Example 7

A negative kaon collides with a proton, a positive kaon and another particle are created. What is the other particle.

Solution

$K^- + p \rightarrow K^+ + x,$ where x is the unknown particle formed.

Applying charge conservation on both side we get

$C: \quad -1 + 1 = +1 + x$

or $x = -1$

This implies that x is negatively charged.

Since kaon is a strange particle, strangeness quantum number must be conserved.

i.e., $S: \quad -1 + 0 = +1 + x$

or $\quad x = -2$

This the strangeness of x is –2. Hence the particle must be Ξ^-

Example 8

Name the conservation law that would be violated in each of the following decays.

a) $\pi^+ \rightarrow e^+ + \gamma$

b) $\Lambda^\circ \rightarrow n + \gamma$

c) $\Omega^- \rightarrow \Sigma^- + \pi^\circ$

d) $\Lambda^\circ \rightarrow \pi^- + \pi^\circ$

Solution

a) $\qquad \pi^+ \rightarrow e^+ + \gamma$

$\quad L: \; 0 \rightarrow -1 + 0 \qquad$ Lepton number not conserved

$\quad B: \; 0 \rightarrow 0 + 0 \qquad$ Baryon number conserved

$\quad S: \; 0 \rightarrow 0 + 0 \qquad$ Strangeness conserved

b) $\qquad \Lambda^\circ \rightarrow n + \gamma$
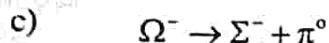
$\quad L: \; 0 \rightarrow 0 + 0 \qquad$ Lepton number conserved

$\quad B: \; 1 \rightarrow 1 + 0 \qquad$ Baryon number conserved

$\quad S: \; -1 \rightarrow 0 + 0 \qquad$ Strangeness number not conserved

c) $\Omega^- \rightarrow \Sigma^- + \pi^0$
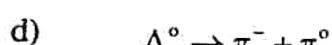
$L:\ 0 \rightarrow 0 + 0$ Lepton number conserved

$B:1 \rightarrow 1 + 0$ Baryon number conserved

$S:-3 \rightarrow -1 + 0$ Strangeness number not conserved

$\therefore \quad \Delta S = -1 - -3 = +2$ Since allowed one for e.m interaction is $\Delta S = 0$

d) $\Lambda^0 \rightarrow \pi^- + \pi^0$

$L:\ 0 \rightarrow 0 + 0$ Lepton number conserved

$B:1 \rightarrow 0 + 0$ Baryon number not conserved

$S:\ -1 \rightarrow 0 + 0$ Since $\Delta S = 1$ and this being a weak interaction it is conserved.

Particle interactions and decays

In this section we discuss how the elementary particles are produced in laboratories and how are they measured. This is to study the structure and properties of elementary particles which is the ultimate aim of particle physics.

The study of atoms and molecules are relatively simple, since they are easily available non-violently. However, the elementary particles, most of which are unstable and do not exist in nature, must be created in violent collisions. For this we require a high energy beam of particles and a suitable target of elementary particles. The only strongly interacting, stable particle is the proton. This is easily available from hydrogen gas. This acts as the garget particles. To get reasonable density of protons researchers use liquid helium rather than gaseous hydrogen.

The incident beam must be accelerated to high energies for the collision to take place. The acceleration process takes place over a long time. so the incident beam particles must be stable. Usually electromagnetic fields are used to accelerate the particles, thus the particle must be charged. A best choice of charged and stable particle is again a proton. So protons are used as incident beam.

In laboratories the high energy incident beam (protons) is allowed to collide with high density target (protons).

The reaction occurs and other types of elementary particles are produced. The reaction is represented as

$p + p \rightarrow$ product particles.
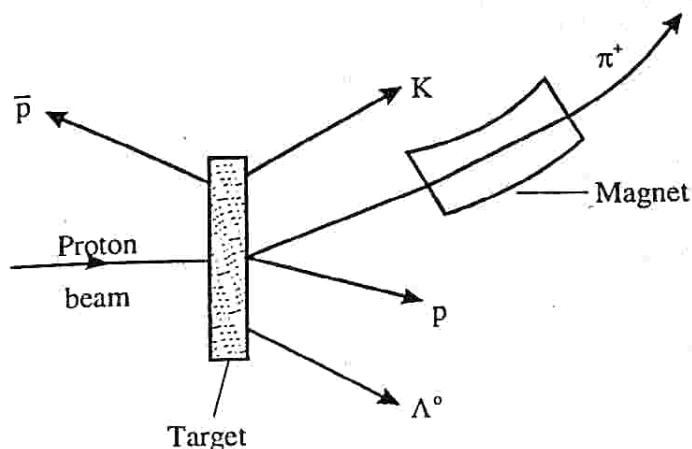
**Figure 5.1: Proton - proton collision process**
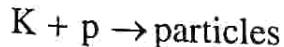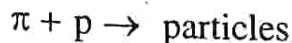
Among the product particles there may exist mesons, baryons, leptons, neutrinos etc. The magnet is used to separate out the desired particles. The conservation laws restrict the nature of product particles. To study one of the particles of product particles, we focus that particle and extract its beam out and made to fall on a second target. The second target might be tens of metres away. For the second reaction to take place the secondary beam must travel tens of metres. The time taken for this is

$$t = \frac{x}{v} = \frac{10m}{3\times10^8 m/s} \simeq 10^{-7}s.$$ This shows that the particle that we selected must have

a life time greater than $10^{-7}s$. But most of the life time of elementary particle is less than $10^{-7}s$. So a secondary reaction seems to be not possible. But relativistic speed of incident beam finds a solution to this. At relativistic speed time dilation effect comes into play. The life time of particle is measured in its rest frame (proper frame), while we are observing the particles flight in the laboratory frame. The time in the laboratory frame might be hundreds of times longer than the proper life time. Even if this proper life time of the particle is $10^{-10}s$. It comes about greater than $10^{-7}s$ in our frame that makes the second reaction possible.

The reaction is

$$\pi + p \rightarrow \text{ particles}$$
$$K + p \rightarrow \text{particles}$$

This reactions enable us to study π and K particles.

Detecting particles

Observing the product particles in the reactions, which may involve dozens of

high energy charged and uncharged particles, and their measurements is a challenge to experimentalists. What they do is a large number of detectors are placed surrounding the reaction area. The particles are recorded by these detectors, no matter what direction they travel after reaction. The particles produce visible tracks in the detector, so that their identity and direction of travel can be determined. The detector provides sufficient mass to stop the particles and measure their energy. A magnetic field is used to curve the charged particles. From the radius of curved path its momentum

$(p=qBr)$ and its energy $\left(\dfrac{q^2B^2r^2}{2m}\right)$ can be calculated. If the detector is a bubble chamber

large number of tracks can be seen. A bubble chamber is a large tank filled with liquid hydrogen in which the passage of a charged particle cause microscopic bubbles resulting from the ionisation of the hydrogen atoms. The bubbles can be illuminated and photographed to reveal the tracks. A photograph of the bubble chamber tracks is given below in figure (5.2). Figure (5.3) is a diagram indicating the particles that participate in the reaction
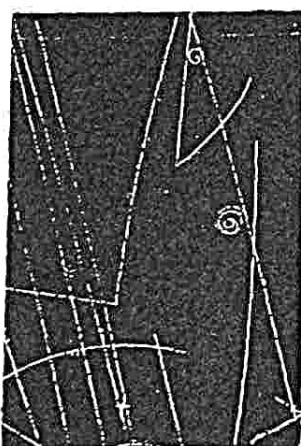
Figure 5.2: Bubble chamber photograph of
   particle reactions
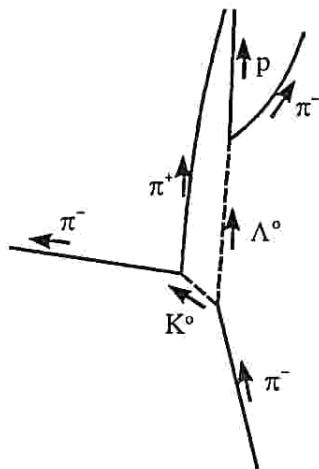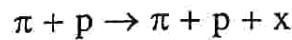
Figure 5.3: Directions of product particles

From careful analysis of the paths of the particles, the mass, momentum and energy of the particles can be deduced. The mean life of the particles can be calculated from the lengths of their tracks by knowing its speeds. Neutral particles leave no tracks. Since the neutrons are unstable and decay into charged particles leaving tracks in the bubble chamber. From these track lengths the mean lifetime of neutral particle can be estimated. ·

The above method of finding mean lifetime works well if the lifetime is of the order of 10^{-10} s or so. This can be extended down to about 10^{-16} s. But many of our
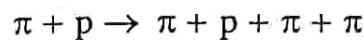
particles have lifetimes of only 10^{-23} s , and a particle moving even at the speed of light the length of the track would be only about $(x = vt)$ 10^{-15} m, the size of a nucleus. How can we measure such a lifetime? Further, how do we even know such a particle exists at all?

Consider the reaction

$$\pi + p \rightarrow \pi + p + x$$

where x is an unknown particle of lifetime 10^{-23} s, which decays into two pions according $x \rightarrow \pi + \pi$

How do we distinguish this reaction from the above reaction

$$\pi + p \rightarrow \pi + p + \pi + \pi$$

which leads to the same particles as actually observed in the laboratory.

Experimental evidence suggests that the two pions in this reaction may combine for an instant $(10^{-23}$ s $)$ to form an entity with all of the usual properties of a particle – a definite mass, charge, spin, lifetime etc. These states are known as resonance particles. The existence of the particles can be inferred indirectly.

Resonance particles

The word resonance implies something in tune or in harmony with something else. In nuclear physics the term resonance is taken from the phenomenon of neutron resonances.

A monoenergetic neutron beam from a reactor is allowed to fall on a target nuclei. Behind the target a neutron detector is placed. By measuring the neutron counts without and with the target present. We can get an idea of the extend to which neutrons of energy are absorbed by the target material.

Suppose we repeat this experiment at various neutron energies and represent the results as a plot of the absorption cross section σ_a of the nuclei in the target and neutron energy, we get a curve as shown in figure 5.4.

The peak at E_0 in σ_a is referred
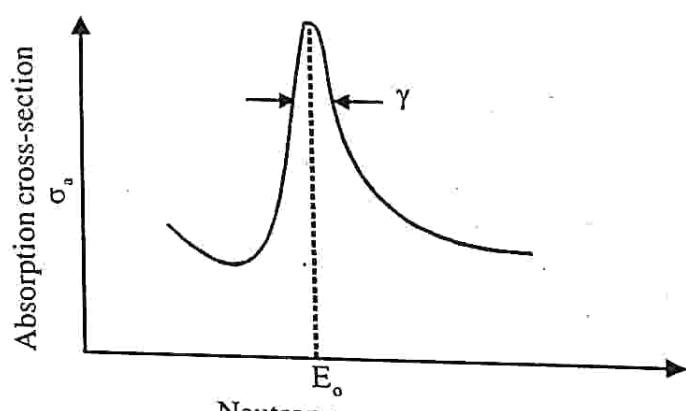
Figure 5.4

to as a neutron capture resonance. Neutrons of this particular energy E_0 are preferentially absorbed by the target nuclei, as compared to neutrons of other energies.

Now we come to the elastic scattering of pions by protons (just mentioned above) and the production of resonant particles. In the early 1950's Fermi was doing experiments on pion-proton, scattering cross section rises as shown in figure indicating the signature of resonance. After performing series of experiments in 1952 Fermi conclusively proved the existence of the phenomenon of resonance. When a pion is bombarded with a target nuclei proton, the target ejects a splatter of new just-born particles. However the lifetime of these particles is very low (10^{-23} s). The two particles (the incoming particle and the target nuclei) temporarily from a bound state like a Bohr atom. But this bound state is short lived. From the excited state the particle returns to normal state by decaying into some other particles through strong interaction. Formerly this bound state was called as resonant states. Later on it was found that mass, spin, magnetic moment etc. can be assigned to the resonant states. After that resonant states are renamed as resonant particles. Now we have large number of resonant particles.

Examples

1. When a proton is bombarded with a pion, at about 1238 MeV the resonant particle is formed (see figure below)

$$\pi^+ + p \rightarrow \Delta^{++}$$

$$\pi^- + p \rightarrow \Delta^0$$
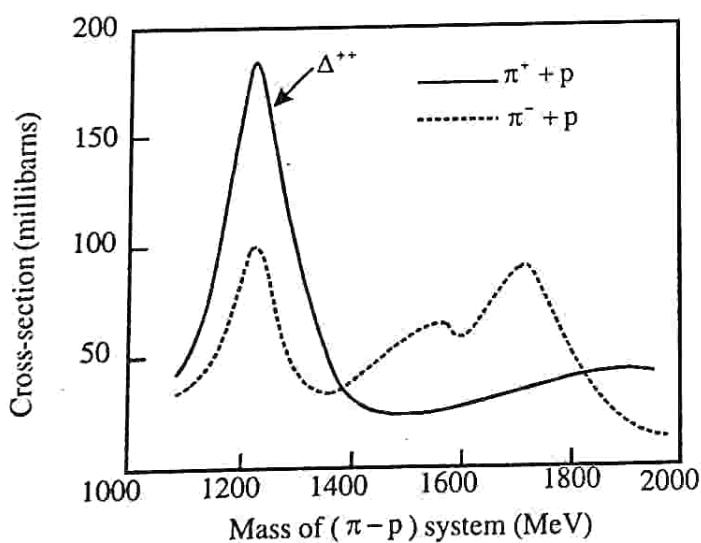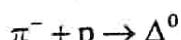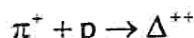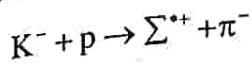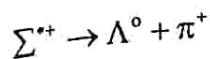
Figure 5.5
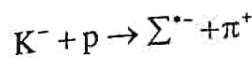
2. When a proton is bombarded with a kaon at about 1385 MeV the resonant particle Σ^* is formed

$$K^- + p \rightarrow \Sigma^{*+} + \pi^-$$
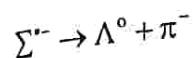
Σ^{*+} decays in 10^{-23} s into

$$\Sigma^{*+} \rightarrow \Lambda^\circ + \pi^+$$

Other possibility is

$$K^- + p \rightarrow \Sigma^{*-} + \pi^+$$
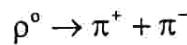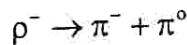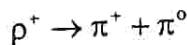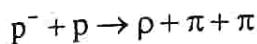
The resonant particle Σ^{*-} decays in 10^{-23} s into

$$\Sigma^{*-} \rightarrow \Lambda^\circ + \pi^-$$

Thus we see that $\Lambda^\circ \pi^+$ and $\Lambda^\circ \pi^-$ resonances are formed

3. Another resonant particle is the rho (ρ) meson. It is a two pions resonance particle. It can exist in three charge states ρ^+, ρ^- and ρ°. These are produced in high energy antiproton-proton annihilations.

$$p^- + p \rightarrow \rho + \pi + \pi$$

$$\rho^+ \rightarrow \pi^+ + \pi^\circ$$

$$\rho^- \rightarrow \pi^- + \pi^\circ$$

$$\rho^\circ \rightarrow \pi^+ + \pi^-$$

Resonances are divided into two categories. They are meson resonances and baryon resonances. Lepton cannot take part in the strong interactions.

Energetic of particle decays

While studying nuclear decays and reactions we used several conservation laws such as linear momentum, energy, angular momentum, charge, lepton number, baryon number, strangeness number etc. Many of these laws can be applied to the analysis of decays and reactions of the elementary particles. But there is a difference between the two. In the production of elementary particles, we require quite high energy for the incident particles. After collision the product particles (elementary particles) move with large velocities. So we have to use relativistic equations for momentum and energy. The production of elementary particles involve high energy, thus the name high energy physics.

The decays of elementary particles can be analysed in a way similar to the decays of nuclei, following the same two basic rules.

1. The Q-value of the decay is

$$Q = (m_i - m_f)c^2$$

Where $m_i c^2$ is the rest energy of the initial particle and $m_f c^2$ is the total rest energy of all the final product particles. It may be recalled that, the decay will occur only if Q is positive.

2. The available energy Q is shared as kinetic energy of the decay products in such a way as to conserve linear momentum. Remember that this kinetic energy is the relativistic one.

i.e., $K = E - m_p c^2 = \sqrt{c^2 p_p^2 + m_p^2 c^4} - m_p c^2$

where m_p is the rest mass of one of the product particles.

Example 9

Find the Q value of the decay

$$\Sigma^+ \to p + \pi^\circ, \quad m_\Sigma = 1189 \frac{MeV}{c^2}, \quad m_p = 938 \frac{MeV}{c^2} \quad \text{and} \quad m_\pi = 135 MeV/c^2$$

Solution

The Q-value is given by

$$Q = (m_i - m_f)c^2$$

$$Q = (m_\Sigma - m_p - m_\pi)c^2$$

$$Q = 1189 - 938 - 135 = 116 MeV$$

Example 10

A neutral Kaon (K°) with a kinetic energy of 276 MeV decays in flight into $\pi^+ + \pi^-$ which move off at equal angles with the original direction of the K°. Find the energies and directions of motion of the π^+ and π^-. $m_{k^\circ} = 498 MeV/c^2$, $m_{\pi^+} = 140 MeV/c^2$

Solution

Using law of conservation of momentum.

Momentum before decay along x-direction

= Momentum after decay along x-direction

$$p_{K^o} = p_{\pi^+} \cos\theta + p_{\pi^-} \cos\theta \qquad \ldots\ldots (1)$$

Along y-direction, we have

$$0 = p_{\pi^+} \sin\theta - p_{\pi^-} \sin\theta$$

or

$$p_{\pi^+} = p_{\pi^-} \qquad \ldots\ldots (2)$$

From the law of conservation of energy, we have

$$E_{K^o} = E_{\pi^+} + E_{\pi^-} \qquad \ldots\ldots (3)$$

$$E_{K^o} = K_{K^o} + m_{K^o}c^2 = 276\text{MeV} + 498\text{MeV} = 774\text{MeV}$$

$$E_{\pi^+} = p_{\pi^+}^2 c^2 + m_{\pi^+}^2 c^4 \qquad \ldots\ldots (4)$$

$$E_{\pi^-} = p_{\pi^-}^2 c^2 + m_{\pi^-}^2 c^4$$

since $p_{\pi^+} = p_{\pi^-}$, we get $E_{\pi^+} = E_{\pi^-}$

From eqn. (3), we get

$$E_{K^o} = 2E_{\pi^+}$$

or $E_{\pi^+} = E_{\frac{K^o}{2}} = \dfrac{774}{2}\text{MeV} = 387\text{MeV}$

From eqn.(4), we get

$$p_{\pi^+}^2 c^2 = E_{\pi^+}^2 - m_{\pi^+}^2 c^4$$

$$p_{\pi^+}c = \sqrt{E_{\pi^+}^2 - (m_{\pi^+}c^2)^2}$$

$$p_{\pi^+}c = \sqrt{(387)^2 - (140)^2} = 360.8\text{MeV}$$

Similarly

$$p_{K^o}c = \sqrt{E_{K^o}^2 - (m_{K^o}c^2)^2}$$

$$p_{K^o}c = \sqrt{(774)^2 - (498)^2} = 592.5\text{ MeV}$$
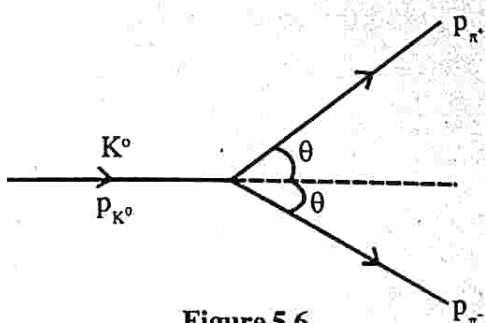
Figure 5.6
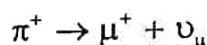
Putting the values of P_{K° and P_{π^+} in eqn. (1), we have

$$\cos\theta = P_{\frac{K^\circ}{2P_{\pi^+}}} = \frac{592.5}{2\times360.8} = 0.8210$$

$\therefore \qquad \theta = 34.81^\circ$

Example 11

Find the kinetic energy of the product particles in the given decay process.

$$\pi^+ \rightarrow \mu^+ + \upsilon_\mu$$

Assume that the decaying particle is at rest.

$$m_\pi = 140\frac{\text{MeV}}{c^2}, \quad m_\mu = 105.7\frac{\text{MeV}}{c^2}$$

Solution
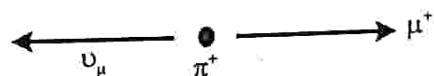
Figure 5.7

From the law of conservation of momentum we have

$$P_\upsilon = P_\mu$$

Energy conservation gives, $E_\pi = E_\mu + E_\upsilon$ ..... (1)

$$m_\pi c^2 = \sqrt{p_\mu^2 c^2 + (m_\mu c^2)^2} + p_\upsilon c$$

or $\qquad m_\pi c^2 - p_\upsilon c = \sqrt{p_\mu^2 c^2 + (m_\mu c^2)^2} \qquad\qquad (\because \ p_\mu = p_\upsilon)$

squaring on both sides, we get

$$(m_\pi c^2)^2 + p_\mu^2 c^2 - 2m_\pi p_\upsilon c^3 = p_\mu^2 c^2 + (m_\mu c^2)^2$$

or $\qquad (m_\pi c^2)^2 - 2m_\pi p_\upsilon c^3 = (m_\mu c^2)^2$

or $\qquad 2m_\pi p_\upsilon c^3 = (m_\pi c^2)^2 - (m_\mu c^2)^2$

$$\therefore \quad p_\upsilon c = \frac{(m_\pi c^2)^2 - (m_\mu c^2)^2}{2m_\pi c^2}$$

$$p_\upsilon c = \frac{(140)^2 - (105.7)^2}{2 \times 140} = 30.1 \text{ MeV}$$

From eqn. (1), we have

$$E_\mu = E_\pi - E_\upsilon = E_\pi - p_\upsilon c$$

$$E_\mu = 140 - 30.1 = 109.9 \text{ MeV}$$

Using $\quad E_\mu = K_\mu + m_\mu c^2$
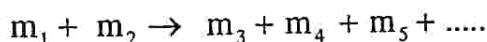
$$\therefore \quad K_\mu = E_\mu - m_\mu c^2 = 109.9 - 105.6$$

$$K_\mu = 4.3 \text{ MeV}$$

Energetics of particle reactions

In particle reaction experiments we found that a beam of high energy particles collides with a target particle at rest results in the production of product particles. Here our aim is to study the kinetimatics of the reaction process. Since the kinetic energies of the particles are usually high, we must go for relativistic formulae. To analyse the reaction process we need some relationships such as Q – value of the reaction and threshold energy to initiate the reaction. Here we are going to derive that.

Consider the reaction:

$$m_1 + m_2 \rightarrow m_3 + m_4 + m_5 + \dots$$

m's stand for masses as well as for particles. Here m_1 is the incident particle which has total energy E_1 and kinetic energy $K_1 = E_1 - m_1 c^2$ and momentum

$$p_1 c = \sqrt{E_1^2 - m_1^2 c^4}, \quad (\because E_1 = \sqrt{p_1^2 c^2 + m_1^2 c^4})$$

The target particle m_2 is at rest, hence having only rest energy $m_2 c^2$. Remember that all are measured in the lab frame of reference.

The Q-value of the reaction is

$$Q = (m_i - m_f)c^2 = \left[(m_1 + m_2) - (m_3 + m_4 + m_5 + \dots) \right]c^2$$

If Q is positive, rest energy is turned into kinetic energy, so that the product particles m_3, m_4, m_5, ... have m are combined kinetic energy than the initial particles m_1 and m_2. If Q is negative some of the initial kinetic energy of m_1 is turned into rest energy.

Threshold energy

When the Q-value is negative, there is a minimum kinetic energy that m_1 must have in order to initiate the reaction is called threshold energy. In the case of non-relativistic nuclear reaction we found that the threshold kinetic energy

$$K_{th} = -Q\left[1 + \frac{m(x)}{m(X)}\right]$$ in the reaction process $x + X \to y + Y$. This K_{th} is larger

than Q. The value is the energy necessary to create the additional mass of the product particles, as well as to impart sufficient kinetic to the product particles to conserve momentum.

Expression for threshold energy

A particle of mass m_1 moving with momentum p_1 and energy E_1 collides a particle of mass m_2 at rest having energy $E_2 = m_2c^2$. After collision large number of particles of masses m_3, m_4, m_5,..... are produced. We assume that all these particles move along a straight line with the same speed together. At threshold this is the most efficient way to provide momentum to the final particles.

Let M and P_M be the total mass and total momentum of the product particles respectively.

According to law of conservation of energy

$$E_1 + E_2 = E_M$$

where E_M is the total energy of the product particles.

i.e., $$\sqrt{(p_1c)^2 + (m_1c^2)^2} + m_2c^2 = \sqrt{(p_Mc)^2 + (m_Mc^2)^2} \qquad ..... (1)$$

According to law of conservation of momentum

We have $p_1 = p_M$

Thus eqn. (1) becomes

$$\sqrt{(p_1c)^2 + (m_1c^2)^2} + m_2c^2 = \sqrt{(p_1c)^2 + (Mc^2)^2}$$

$$(\because p_M = p_1 \text{ and } m_M = M)$$

squaring on both sides, yields

$$(p_1c)^2 + (m_1c^2)^2 + (m_2c^2)^2 + 2m_2c^2\sqrt{(p_1c)^2 + (m_1c^2)^2}$$

$$= (p_1c)^2 + (Mc^2)^2$$

or

$$(m_1c^2)^2 + (m_2c^2)^2 + 2m_2c^2\sqrt{(p_1c)^2 + (m_1c^2)^2} = (Mc^2)^2$$

i.e.,

$$= \sqrt{(p_1c)^2 + (m_1c^2)^2} = \frac{(Mc^2)^2 - (m_1c^2)^2 - (m_2c^2)^2}{2m_2c^2}$$

The threshold kinetic energy of m_1 is

$$K_{th} = E_1 - m_1c^2$$

$$K_{th} = \sqrt{(p_1c)^2 + (m_1c^2)^2} - m_1c^2$$

$$K_{th} = \frac{(Mc^2)^2 - (m_1c^2)^2 - (m_2c^2)^2}{2m_2c^2} - m_1c^2$$

$$K_{th} = \frac{(Mc^2)^2 - (m_1c^2)^2 - (m_2c^2)^2 - 2m_1m_2c^4}{2m_2c^2}$$

$$K_{th} = \frac{(Mc^2)^2 - (m_1c^2 + m_2c^2)^2}{2m_2c^2}$$

$$K_{th} = \frac{(Mc^2 + m_1c^2 + m_2c^2)(Mc^2 - m_1c^2 - m_2c^2)}{2m_2c^2}$$

(Used $a^2 - b^2 = (a+b)(a-b)$)

But $Q = m_1c^2 + m_2c^2 - Mc^2$
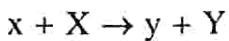
$$K_{th} = \frac{-Q(Mc^2 + m_1c^2 + m_2c^2)}{2m_2c^2}$$

or

$$K_{th} = \frac{-Q[m_1 + m_2 + m_3 + m_4 + m_5...]}{2m_2}$$

i.e.

$$K_{th} = \frac{-Q \times \text{total mass of all particles involved in reaction}}{2 \times \text{mass of target particle}}$$

This is the expression for threshold kinetic energy.

Consider the reaction

$$x + X \rightarrow y + Y$$

$$K_{th} = \frac{-Q[m(x) + m(X) + m(y) + m(Y)}{2m(X)}$$

In the non-relativistic case, all kinetic energies are very small compared to rest energies, so that the total rest energy in the reaction does not change.

i.e., $m(x) + m(X) = m(y) + m(Y)$

so the above equation becomes

$$K_{th} = -\frac{Q}{2} \frac{[2m(x) + 2m(X)]}{m(X)}$$

or

$$K_{th} = -Q\left[1 + \frac{m(x)}{m(X)}\right]$$

This is our expression for non-relativistic threshold kinetic energy. This shows that in the limit of low speeds, the relativistic threshold kinetic energy formula reduces to the non-relativistic formula.

Finally we estimate the efficiency of the reactions. Efficiency in the sense that how much initial energy that we supply actually goes into producing the reaction products (rest energy of the product particles) and how much is wasted as the kinetic energies of the reaction products. Suppose 400 MeV of kinetic energy is given to a reaction to produce 200 MeV of rest energy of the product particles, then efficiency

$$= \frac{200 \text{ MeV}}{400 \text{ MeV}} = \frac{1}{2} = 50\%$$

For example to produce a particle of 50 GeV rest energy, suppose we require 1250 GeV of initial kinetic energy. Then
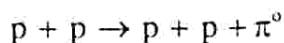
$$\text{efficiency} = \frac{50 \text{ GeV}}{1250 \text{ GeV}} = 0.04 = 4\%$$

This show that efficiency is only 4%, where 96% of initial kinetic energy must go to the kinetic energies of particles. This is obviously an uncomfortable situation for particle physicists. This is because if they try to produce more massive particles, they must build more powerful accelerators.

To overcome this difficulty, we do the experiments in C M frame. In this frame two particles with equal and opposite momentum collide head-on. After collision particles produced are at rest in the C M frame. That means no initial kinetic energy is transferred to the kinetic energy of the product particles. Thus the efficiency of this reaction is 100%. For example two protons of 25 GeV kinetic energy collide head-on the total energy after collision is 50 GeV. This can produce a particle of 50 GeV rest energy. This is the mechanism used in large hadron collider (LHC) that we already discussed. To have more clarity about efficiency see examples 12 and 13.

Example 12

Find the threshold kinetic energy and efficiency of the reaction.

$$p + p \rightarrow p + p + \pi^\circ$$

$$m_p = 938 \frac{\text{MeV}}{c^2} \qquad m_\pi = 135 \frac{\text{MeV}}{c^2}$$

Solution

$$Q = (m_i - m_f)c^2 = [m_p + m_p - (m_p + m_p + m_\pi)]c^2$$

$$Q = - m_\pi c^2 = -135 \text{MeV}$$

Threshold kinetic energy

$$K_{th} = -Q \frac{\text{Total mass of all particles involved}}{2 \times \text{mass of the target}}$$

$$K_{th} = -\frac{Q(4m_p + m_\pi)}{2m_p}$$
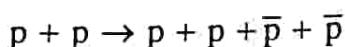
$$K_{th} = \frac{--135\,(4\times938+135)}{2\times938}$$

$$K_{th} = 279.7 \text{MeV}$$

$$\text{Efficiency} = \frac{\text{Rest energy of } \pi^o \text{ produced}}{\text{Threshold kinetic energy}}$$

$$= \frac{135}{279.7} = 0.4820 = 48.2\%$$

Example 13

Calculate the threshold kinetic energy and efficiency of the reaction

$$p + p \rightarrow p + p + \bar{p} + \bar{p}$$

Solution

$$Q = (m_i - m_f)c^2$$

$$Q = [2m_p - (2m_p + 2\bar{m}_p)]c^2$$

$$Q = -2\bar{m}_p c^2$$

The rest energy of the antiproton ($\bar{p}$) is the same as that of the proton

$$\therefore \quad Q = -2\bar{m}_p c^2$$

Threshold kinetic energy:

$$K_{th} = \frac{-Q \text{ Total mass of all products involved}}{2 \times \text{mass of the target}}$$

$$K_{th} = --2m_p c^2 \frac{6m_p c^2}{2m_p c^2} = 6m_p c^2$$
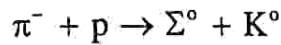
$$K_{th} = 6\times938\text{MeV} = 5628\text{MeV}$$

$$\text{Efficiency} = \frac{\text{Rest energy of particle produced}}{\text{Threshold kinetic energy}}$$

$$= \frac{2m_p c^2}{6m_p c^2} = \frac{1}{3} = 33.33\%$$

Example 14

Find the threshold kinetic energy for the reaction in the lab frame.

$$\pi^- + p \rightarrow \Sigma^\circ + K^\circ$$

$$m_{\pi^-} = 140 \frac{MeV}{c^2}, \quad m_p = 938 \frac{MeV}{c^2}, \quad m_{\Sigma^\circ} = 1192 \frac{MeV}{c^2} \text{ and } m_{K^\circ} = 498 \frac{MeV}{c^2}$$

Solution

$$Q = (m_i - m_f)c^2 = (m_\pi + m_p - m_\Sigma - m_K)c^2$$

$$Q = (140 + 938 - 1192 - 498)MeV$$

$$Q = -612 \text{ MeV}$$

$$K_{th} = -Q \frac{[m_\pi + m_p + m_\Sigma + m_K]}{2m_p}$$

$$K_{th} = --612 \frac{[140 + 938 + 1192 + 498]}{2 \times 938}$$

$$K_{th} = 902.99 \text{ MeV}$$

The quark model

Quarks

What are the basic building blocks of matter? We had been trying to answer this question for a long time. In the last session we arrived at a conclusion that the elementary particles discussed are the basic building blocks of matter. Electrons, protons, neutrons, pions, kaons, photons, gravitons, resonant particles etc. were termed as fundamental or elementary particles. The developments and researches in the theoretical physics predicted particles which are more fundamental than elementary particles discussed in the last session. These particles are named as quarks. In 1964 Murry Gell-mann and independently G zweig introduced the idea of the basic unit named quarks to explain the internal constitution of baryons and mesons. Zweig called his particles 'ace' but it was the same as the quark of Gell-mann. The quark model was not a theoreticians mathematical artifice. It was actually being demanded by experimentalist and their experiments. Upto about 50 years ago it was thought that protons and neutrons were elementary particles, but experiments in which protons were collided with other protons and electrons at high speeds indicated that they

were in fact made up of smaller particles. These particles were named as quarks. Gell-mann taken this name from a song in the novel 'Finnigans wake' written by James Joyce. The song goes like this:

Three quarks for muster mark !

Sure he hasn't got much of a bark

And sure any he has its all beside the mark.

Joyce used the term 'quark' for the cry of a frog (crok- crok)

In the quark model of Gellmann he developed theoretically the following attributes to the quarks.

(i)   All the three quarks (u, d and s) have fractional charges.

up quark has a charge $\frac{2}{3}e$

down quark has a charge $-\frac{1}{3}e$

strange quark has a charge $-\frac{1}{3}e$

(ii) All quarks have spin $\frac{1}{2}\hbar$

i.e., quarks are fermions

(iii) The antiparticle of quarks are called antiquarks. $\bar{u}$, $\bar{d}$ and $\bar{s}$ are the antiquarks of up, down and strange quarks respectively.

(iv) All baryons are made up of three quarks. For example proton (baryon) is made up of u, u and d quarks.

i.e., $u + u + d = p$

$$\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = e$$

So charges are conserved.

A neutron is made up of u, d, d quarks

i.e., $u + d + d = n$

$$\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$$

Similarly an antiproton is made up of $\bar{u}, \bar{u}$ and $\bar{d}$

i.e., $\qquad \bar{u} + \bar{u} + \bar{d} = \bar{p}$

$$-\frac{2}{3}e + \frac{-2}{3}e + \frac{1}{3}e = -e$$

An antineutron is made up of $\bar{u}, \bar{d}$ and $\bar{d}$

i.e., $\qquad \bar{u} + \bar{d} + \bar{d} = \bar{n}$

$$\frac{-2}{3}e + \frac{1}{3}e + \frac{1}{3}e = 0$$

V. All mesons are made of one quark and one antiquark.

Example

1. $\pi^+ = u + \bar{d}$

2. $K^+ = u + \bar{s}$

We can easily check the conservation of charges on both sides.

Gellmanns quarks and their attributes

Name	Symbol	Rest mass in MeV	Charge in units of e	spin $\hbar$
Up	u	310	$\frac{2}{3}$	$\frac{1}{2}$
Down	d	310	$-\frac{1}{3}$	$\frac{1}{2}$
Strange	s	505	$\frac{-1}{3}$	$\frac{1}{2}$

Using the above six rules Gellmann could be readily explained all the known particles in the mid sixties. By using quark model Gell-mann formulated a method of classifying elementary particles that enabled him to predict omega ($\bar{\Omega}$) particle which is made of three s quarks

i.e., $\qquad\qquad \bar{\Omega} = s + s + s$

Later $\bar{\Omega}$ particles was experimentally detected, it was a great truimph for Gell-mann and his quarkmodel. For this Gellmann was awarded the Nobel prize in physics in 1969.

Flavour of quarks

In addition to u,d, and s quarks three more quarks were predicted and the signature of their existence were detected. They are named as charm (c), top (t) and bottom (b) quarks. Earlier the t and b quarks used to be called truth and beauty respectively, but some how, the mundane labels top and bottom have replaced them. Now in the quark family there are six members and their corresponding antiparticles. One can either think of it as six different quarks or six dfferent states of the same particle just as the neutron and the proton can be regarded as two different states of the nucleon. In the latter case, one refers to the six different quarks states as having different flavours. Considering quarks and antiquarks together one can say there are 12 flavours of quark and antiquark.

Name	Symbol	Restmass in MeV	Charge in units of e	Spin $\hbar$
Charm	c	1500	$\dfrac{2}{3}$	$\dfrac{1}{2}$
Top	t	177000	$\dfrac{2}{3}$	$\dfrac{1}{2}$
Bottom	b	5000	$-\dfrac{1}{3}$	$\dfrac{1}{2}$

Colour of quarks

Gell-manns original quark model predicted only three quarks namely u, d, and s. Even at that time, a serious difficulty was noticed in the Gell-mann's model. To understand this let us consider the omega (Ω^-) hyperon. It is made of three s quarks

i.e., $\Omega^- = s + s + s$

The spin $\dfrac{3}{2}\hbar$ of Ω^- is easily understood by noting that it has three quarks (sss), each of which has a spin of $\dfrac{1}{2}\hbar$. Similarly the charge of Ω^- is negative since each s has a charge $-\dfrac{1}{3}e$. But then quarks being fermions how can one have more than one quark in the same spin state. According to Paulis exclusion principle no two fermions can be in the same state. In other words quark model is violating the Paulis exclusion principle. This was the problem but a solution was quickly found borrowing upon earlier experience in atomic and nuclear physics. The trick was to introduce another

property for the quarks so that the three s quarks in Ω^-, while all being in the spin $\frac{1}{2}\hbar$ state differ with respect to the property we assign to them. Let us call this new property C and let us suppose that C can take on three values C_1, C_2 and C_3, Just as spin $\frac{1}{2}\hbar$ can take values $\frac{1}{2}\hbar$ and $-\frac{1}{2}\hbar$. With the aid of C we can now stipulate that the three s quarks in Ω^- should occupy the states $\left(\frac{1}{2}\hbar, C_1\right)$, $\left(\frac{1}{2}\hbar, C_2\right)$ and $\left(\frac{1}{2}\hbar, C_3\right)$. Obviously the states of s are different.

The idea of invoking such a new property was first proposed by Oscar green berg and independently by Nambu who called this new property colour. According to Nambu each quark comes in three colours. Red (R), green (G) and blue (B). Thus according to this idea in Ω^- one of the quarks is red, the second one is green and the third one is blue. What about the antiquarks? What are their colours. Nambu said that the antiquarks have anticolours. i.e., antired $(\bar{R})$, antigreen $(\bar{G})$ and anti blue $(\bar{B})$.

It must be noted that this colour has nothing to do with colour in every day parlance. This new colour property was assigned to quark to maintain the status quo of the well established (experimentally) Paulis exclusion principle. According to the colour property assigned to quarks, all three quarks in a baryon have different colours which satisfies the Paulis exclusion principle. Such a combination can be thought of as while by analogy with the way red, green and blue light combine to make white (colourless). Similarly an antibaryon consists of an antired, an anti green and an antiblue quark. A meson consists of a quark of one colour and an anti quark of the corresponding anti colour, which has the effect of cancelling out the colour. The results is that hadrons (baryons and mesons) and antihandrons are colourless.

Note: To have some idea about anti colours in analogy with the colours that one can perceive with the human eyes go through the following.

Since all baryons made up of three quarks, are colour less, we must have three quarks with different colours

$$R + G + B = W \text{ (white)} \qquad .....(1)$$

All mesons, made up of a quark and an antiquark, are colour less. We must have

$$R + \bar{R} = W \qquad .....(2)$$

comparing equation (1) and (2) we have $\bar{R} = G + B$

i.e., combing green and blue, we get antired. Combining blue and green we know that we get cyan (peacock blue)

It means that antired is nothing but cyan similarly we have

$$R + G + B = W$$

and

$$G + \bar{G} = W$$

Comparing

$$\bar{G} = R + B$$

i.e., combining red and blue, we get antigreen. Red and blue combines to magenta. It means that anti green is nothing but magenta.

Using

$$R + G + B = W$$

and

$$B + \bar{B} = W$$
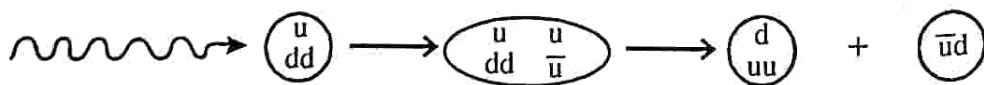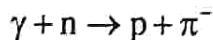
comparing

$$\bar{B} = R + G$$

Antiblue is the combination red and green. Combining red and green we get yellow it means that antiblue is nothing but yellow.

Quark confinement

When the quarks were first proposed, the first question people asked was how can we see quarks. After that people were in the hot pursuit of conducting experiments for the detection of free quarks. None could be successful. Finally they concluded that quarks were permanently imprisoned inside. This is the idea of quark confinement.

With sufficient energy one can tear an atom apart and free the electrons and nucleus inside. With even more energy one could tear a nucleus into its constituent neutrons and protons. But with still higher energy one could not manage to knock out a quark from a baryon or a meson. When one hits a baryon very very hard something must happen. It does so but quarks manage to stay inside and never comes out singly even if others are produced. When enough energy is given, instead of a quark breaking free from the others in a hadron, the excess energy goes into producing a quark-antiquark pair. this results in a meson that does escape. For example when an energetic gamma ray photon impinges on a neutron (udd) and causes a $u\bar{u}$ quark-antiquark pair to come into being. The quarks $udd + u\bar{u}$ then rearrange themselves into a proton (uud) and negatively charged pion ($\bar{u}d$). The net reaction is as follows

$$\gamma + n \rightarrow p + \pi^-$$

The quark pair is like a chest expander whose spring is infinitely strong. Consider a chest expander spring. The more one pulls, the harder it is to expand the spring. The more one pulls, the harder it is to expand the spring further. Let us consider the other extreme. If the spring is laid loosely on the ground the two ends can be easily and independently pushed around. This analogy carriers over beautifully into the world of quarks. The spring that binds two quark is colour force and is like the chest expander. The more one increases the distance between two quarks, the harder it becomes to separate them further.

Quarkonium

We found that baryons consist of three quarks. Technically speaking it is a three body quantum mechanical system. As there is no mathematical tool to tackle a three body problem, it is very difficult to extract the properties and interactions of quarks forming baryons. However we can learn a little bit about the interactions of quarks from examining the properties of two body systems, especially quark-anti quark pairs in mesons.

We know that mesons are formed by the combination of quarks and antiquarks. The binding energies of quarks in mesons are very large (hundreds of MeV), so the quark-antiquark pairs of the light quarks (u, d, s) must be treated relativistically. This is because the binding energies and rest energies of light quarks are roughly the same. However, for the massive quarks (c, b, t) the binding energies are small compared to rest energies, so we can use non-relativistic methods for the analysis.

The bound quark-antiquark pair is called quarkonium. We can study the properties of quarkonium in analogy with the well studied positronium. The bound state of positron electron pair is called positronium. The positronium structure is very much similar to quarkonium. In positronium, positron and electron each orbit about their centre of mass. It is very much like a hydrogen atom. Hence we can label the states of positronium with orbital angular momentum quantum number L and spin quantum number S.

The total orbital angular momentum quantum number $L = 0$ for s states and $L = 1$ for p states. If the two particles are parallel their total spin $S = 1$, for antiparallel $s = 0$. The system can exist with different radial wave functions that we can label it by principal quantum numbers n = 1, 2, 3.....

These designations can as such be applied to quarkonium. It has been found that $c\bar{c}$ quarkonium structure is very much similar to positronium. The lowest $S = 1$, $L = 0$, $c\bar{c}$ state is the J/ψ meson. This meson was discovered in the year 1974 and could be explained on the basis of quark structure. Unfortunately the structure of

$b\bar{b}$ quarkonium resulting in γ-meson has not been completely verified in experiments. The only thing that we know is all quarkonium structures similar to positronium.

The knowledge of the excited states of quarkonium enables us to guess a model for their potential energy of interaction. There are several models available among the researchers of quark-gluon plasma. One simple model of potential energy is

$$U = \frac{a}{r} + br$$

By using potential energy, we can solve the Schrodinger equation to get energies of different states. The values of a and b are chosen in such a way that the energy that we obtained from solving Schrodinger equation must tally with experimental result. The constant b turns out to be $GeV(fm)^{-1}$. This shows that large amount of energy is required to separate two quarks apart. To separate quarks in a meson even to a atom sized distance $(10^{-10}m = 10^{5}fm)$, we require about $10^{5}GeV$ energy. This energy is get to be achieved in accelerators. This is the reason why we didn't see free quarks in laboratories. Now we believe that one day this energy will be produced in accelerators so that we can see free quarks which are the basic building blocks of the universe.

The standard model

Introduction

Until the 1930's all natural phenomena were presumed to have their origin in just two basic forces - they were gravitational and electromagnetism. Both were described by classical fields that permeated all space. These fields extended out to infinity from well defined sources, mass in one case and charge in the other. This rule over the physical universe seemed securely established. As atomic and subatomic phenomena were explored, it became apparent that two completely novel forces had to be added to this list, they were the weak and strong force. The strong force was necessary inorder to understand how the nucleus is held together, protons bound together in tight nuclear ball $(10^{-15}m$ size) must be subject to a force much stronger than electromagnetism to prevent their flying apart. The weak force was invoked to understand the transmutation of a neutron in the nucleus into a proton during the particularly slow form of radioactive decay known as beta decay.

Since neither the weak force nor strong force is directly observed in the microscopic world, both must be very short range relative to the more familiar gravitational and electromagnetic forces. Furthermore, the relative strengths of the forces associated with all four interactions are different. $(F_s : F_e : F_w : F_g = 1 : 10^{-2} : 10^{-5} : 10^{-38})$. It

is therefore not too surprising that for a very long period these interactions were thought to be quite separate. Inspite of this, there had always been a lingering suspicion and hope that in some miraculous fashion all four were simply manifestations of one source or principle and could therefore be described by a single unified theory.

The first step towards unification was begun in 1967 with the development of the electroweak theory (EW) by Stephen Weinberg and Abdus Salam. In this theory, the weak and electromagnetic interactions are regarded as separate aspects of the same force - the electroweak force, just as electric and magnetic forces are distinct but part of a single phenomenon, electromagnetism. The theory predicted the existence of vector bosons ($W^{\pm}, Z^{\circ}$) were experimentally detected which provided a confirmation to the electroweak force. After this we have only three basic forces strong, electroweak and gravitational. The next aim of the scientific community was to unify strong and electroweak forces under the name standard model.

According to this model the strong force that binds nucleons into nuclei and is mediated by pion exchange is the external manifestation of the colour force between quarks in the nucleons that is mediated by gluons. That is the strong force between quarks is carried by an exchanged particles called the gluon, which provides the, glue that binds the quarks together in mesons and baryons. There are totally eight different gluons in this model. A theory known as Quantum Chromo Dynamics (QCD) describes the interactions of quarks and exchange of gluons. Like the quarks, gluons have not been observed experimentally, but there is indirect evidence of their existence from a variety of experiments.

The Standard Model (SM)

The standard model is the amalgamation of strong interaction and electroweak interaction. The standard model sees the world as built up of two sets of particles quarks and leptons and the field particles. There are six quarks and 6 leptons and their antiparticles plus the field particles (photons, 3 vector bosons (W^{+}, W^{-} and Z°) and eight gluons). The quarks are described by QCD and leptons by EW theory. The unified theory of these three interactions (strong, weak and electromagnetism) is called the standard model. The gravitational interaction does not come under the standard model.

Though this model is not fully successful, it is on the way to its success. The main drawback of this model that important elements of the model have to be inserted arbitrarily. The model require us to measure the masses of leptons and quarks experimentally. Inorder for the standard model of quarks and leptons to be mathematically consistent, the Scottish physicist Peter Higgs showed that, a field now called the Higgs field must exist everywhere in space. When particles interact with Higgs field they acquire their characteristic mass. The stronger the interaction

the greater the mass, we can think of the Higgs field as exerting a kind of viscous drag on the particles that move through it, this drag appears as inertia, the defining property of mass. As with the other, a particle here called Higgs boson mediate the action of Higgs field.

Discovery of Higgs bosons and knowing its mass and behaviour predicted by standard model would be major step in validating the standard model. For this we constructed a very powerful accelerator at CERN in Switzerland, Geneva called Large Hadron Collider (LHC). We already discussed about LHC in detail. LHC is currently searching for the evidence of the Higgs particles by colliding beams of protons at an energy of 7000 GeV. The physics community is eagerly looking at the observation results going to be revealed.

The second drawback of the standard model is that it is based on massless neutrinos. But experiments revealed that neutrinos are massive even though very small (mass of electroneutrino is 2 eV). So the standard model must be extended to include non zero neutrino masses and the rules for conservation of lepton number must be modified to allow one type of neutrino into another called neutrino oscillation.

Neutrino Oscillation

Measurements of the flux of neutrinos reaching the Earth from the Sun, produced in the fusion reactions, revealed that there is a large defect. The intensity of electroneutrinos observed on Earth is only about 1/3 of what is predicted based on models of how fusion reactions occur in the sun's interior. Recent measurements revealed that the intensity of all neutrions including muon and tau neutrions are in agreement with the predicted rate. At the same time sun produces only electron neutrinos. This is because the reacting particles in the solar interior are not sufficiently energetic to produce muon and tau neutrinos. Then how they come in our atmosphere. This mystery has been explained by proposing that the electron neutrinos are produced in the solar interior at the expected rate, but that during journey from the sun to the Earth, the purely electron neutrinos becomes a mixture of roughly equal parts of electron, muon and tau neutrinos. This explains why only 1/3 of electron neutrinos are present on Earth. This phenomenon of converting electron neutrinos into muon and tau neutrinos is called neutrino oscillation.

The third drawback of the standard model is that the fundamental particles and forces are quarks and eight gluons. They have not observed so far. Furthermore nobody so far has been able to derive the details of strong force emerging from colour quarks.

The unified theory explaining the four interactions is called Grand Unification Theory (GUT).

IMPORTANT FORMULAE

1. Basic forces : Strong, electromagnetic, weak and gravitational

2. Field particles : Gluons, photon, vector bosons (W^+, W^- and Z^o) and graviton

3. Leptons : e^-, υ_e, μ^-, υ_μ, τ^- and υ_τ

4. Mesons : π^+, π^-, π^0, K^+, K^-, K^0, η, ρ^+, ρ^-, η'.......

5. Baryons : p, n, Λ^0, Σ^+, Σ^-, Σ^0, Ξ^-, Ξ^0, Ω^-.........

6. Conservation of lepton number : In any process L_e, L_μ and L_τ remain constant

7. Conservation of baryon number : In any process remains B constant

8. Conservation of strangeness : In strong and electromagnetic process S remains constant, in weak process $\Delta S = 0, \pm 1$

9. Q-value in reactions, $Q = (m_i - m_f)c^2$

10. Threshold kinetic energy in reactions, $K_{th} = \dfrac{-Q(m_1 + m_2 + m_3 + ...)}{2m_2}$

11. Quarks : u, d, c, s, t, b.

12. In high energy physics, relativistic momentum is conserved also total energy $E = K + mc^2$ is also conserved.

Where $E = \sqrt{p^2 c^2 + m^2 c^4}$ and mc^2 is the rest energy.

UNIVERSITY MODEL QUESTIONS

Section A

*(Answer questions in about **two** or **three** sentences)*

Short answer type questions

1. What are elementary particles?
2. Name the four basic forces in nature?
3. Which forces are to be considered while determining the stability and structure of nuclei?
4. How will you distinguish between weak force and strong force?
5. Distinguish between particles and antiparticles?
6. What is antihydrogen atom?
7. Name the three types of neutrinos?
8. Do all strongly interacting particles also feel weak interaction?
9. What are Fermions and Bosons?
10. Classify the elementary particles on the basis of mass.

11. What are baryons? Name three of them?
12. What are mesons? Name three of them?
13. What are leptons? Give their names?
14. What are the types of interactions that leptons take part?
15. What are the types of interactions that baryons take part?
16. Give an example for lepton number conservation?
17. Give an example for baryon number conservation?
18. Give an examples for strangeness conservation?
19. In particle interactions and decays the beam particle and target particle are protons. Why?
20. What are resonance particles?
21. Give an example for the formation of resonance particle?
22. In high energy particle physics, the neutral particles produced in reactions leave no tracks in the bubble chamber, then how will we estimate their lifetime?
23. What is meant by threshold energy in high energy particle reactions?
24. Write down an expression for threshold kinetic energy in particular reactions and explain the symbols used?
25. What is meant by efficiency of a nuclear particle reaction?
26. How do we improve the efficiency of particle reactions?
27. In high energy particle physics, lab frame or CM frame which is more efficient to conduct experiments?
28. What are quarks?
29. Protons and neutrons are made up of quarks. Explain?
30. What is meant by flavour of quarks?
31. Give the names of flavours of quarks?
32. What is meant by colour of quarks?
33. What is standard model?
34. What are Higgs bosons?
35. What is electroweak theory?
36. What are messenger or field particles?
37. Write down the names of all field particles?
38. Write down the decay of a positively charged pion?
39. Write down the decay of a negatively charged pion?
40. What is meant by quark confinement?
41. What is quantum chromodynamics?
42. What is quarkonium?
43. Which are the mediating particles of electroweak force?
44. What is graviton?
45. What is neutrino oscillation?

Section B

*(Answer questions in a **paragraph** of about half a page to one page)*

Paragraph / Problem type questions

1. Distinguish between gravitational force and electromagnetism?
2. What is strong force? Write down its properties?
3. What is weak force? Write down its properties?
4. Neutrinos are not a part of ordinary matter, then how will you distinguish between neutrino and an atineutrino?
5. Distinguish between four basic forces ?
6. Write a brief note on three member families of particles?
7. What is lepton number conservation?
8. What is baryon number conservation?
9. What is strangeness number conservation?
10. The mean life time of most of the product particles is about 10^{-10} s in particle reactions. Then how it is possible to produce a secondary collision where the second target is at a distance of 10m away from the first target
11. How do we detect product particles in particles reactions?
12. How will we calculate the mean life of product particles produced in high energy particle decay?
13. How can we measure the mean life of a particle of the order of 10^{-23} s produced in particle reactions?
14. Explain what is meant by resonance and resonance particles?
15. What are the two basic rules that can be applied in the decay of elementary particles produced in high energy reactions?
16. Show that in the non-relativistic limit, the relativistic expression for threshold kinetic energy becomes non-relativistics threshold kinetic energy?
17. Why quarks have fractional charges? Substantiate with two examples.
18. Explain the emergence of colour of quarks?
19. Explain briefly the standard model?
20. What are the drawbacks of standard model?
21. Explain the origin of neutrino oscillation?
22. Which of the following reactions can occur? State the conservation principles violated by the others.

    a) $p + p \rightarrow n + p + \pi^+$ b) $p + p \rightarrow p + \Lambda^\circ + \Sigma^+$

    c) $e^- + e^+ \rightarrow \mu^+ + \pi^-$ d) $p + p \rightarrow \pi^+ + K^\circ + \Lambda^\circ$

    [a) possible, b) not possible, c) not possible, d) possible]

23. Check whether the given reaction is possible or not

a) $K^+ \to \pi^+ + \pi^+ + \pi^-$ b) $K^+ \to \mu^+ + \upsilon$ [a) not possible, b) not possible]

24. Which one is a possible reaction

a) $\pi^- + p \to \Lambda^\circ + K^\circ$ b) $K^- + p \to \Sigma^+ + \pi^-$ [a) possible, b) possible]

25. Which quarks make up the negative pion?   [$\bar{u}d$]

26. What particle corresponds to the quark composition uus   [Σ^+]

27. Can a Λ particle decay into π^+ and π^- meson   [No]

28. The quantum numbers of three quarks are tabulated below

	Charge	B. No.	Strangeness
q_1	$\frac{2}{3}e$	$\frac{1}{3}$	0
q_2	$-\frac{1}{3}e$	$\frac{1}{3}$	0
q_3	$-\frac{1}{3}e$	$\frac{1}{3}$	-1

construct Σ^- from the above quarks   [$q_2\ q_2\ q_3$]

29. Identify the interaction responsible for the following decays

a) $\Delta^* \to p + \pi \ (10^{-23}s)$ c) $K^+ \to \mu^+ + \upsilon_\mu \ ((10^{-8}s)$

b) $\eta \to \gamma + \gamma ((10^{-18}s)$ d) $\Lambda^0 \to \pi + p \ (10^{-10}s)$

$$\begin{bmatrix} \text{a) Strong} & \text{b) E.M} \\ \text{c) Weak} & \text{d) weak} \end{bmatrix}$$

30. Name the conservation law that would be violated in each of the following

a) $\Lambda^0 \to p + K^-$ c) $\Xi^0 \to \Sigma^0 + \pi^0$

b) $\Omega^- \to \Xi^0 + K^-$ d) $\mu^- \to e^- + \gamma$

Given $m_\Lambda = 1116 \text{ MeV}/c^2$

$m_K = 494 \text{ MeV}/c^2$ $m_{\Xi^0} = 1315 \text{ MeV}/c^2$

$m_{\Sigma^0} = 1193 \text{ MeV}/c^2$ $m_{\pi^0} = 135 \text{ MeV}/c^2$

$$\begin{bmatrix} \text{a) energy} & \text{b) energy} \\ \text{c) energy} & \text{d) } L_e \text{ and } L_\mu \text{ are not conserved} \end{bmatrix}$$

31. Each of the following reactions violates one of the conservation laws. Identify it?
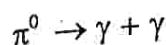
a) $\upsilon_e + p \to n + e^+$ (L_e not conserved)

b) $p + p \to p + n + K^+$ (S not conserved)

c) $\pi^- + n \to K^- + \Lambda^0$ (S not conserved)

32. A Σ^- baryon is produced in a certain reaction with a kinetic energy of 3642 MeV. If the particle decays after one meanlife time, what is the longest possible track this particle could leave in a detector. Proper life time of $\Sigma^- = 1.5 \times 10^{-10}$s and $m_{\Sigma^-} = 1197$ MeV$/c^2$ **[0.18m]**
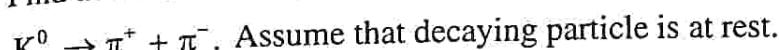
33. Find the Q-value of the following decay?

    $\pi^0 \rightarrow \gamma + \gamma$

    $m_\pi = 135$ MeV$/c^2$ **[135 MeV]**
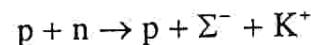
34. Find the kinetic energies of the two product particles in the given decay

    $K^0 \rightarrow \pi^+ + \pi^-$. Assume that decaying particle is at rest.

    $E_{K^0} = 498$ MeV$/c^2$ and $m_{\pi^-} = 148$ MeV $\left[K_{\pi^+} = K_{\pi^-} = 109 \text{ MeV}\right]$

35. A negatively charged sigma particle moving with a kinetic energy of 250 MeV decays into $\pi^- + n$. The π^- moves at $90°$ to the original direction of travel of the Σ^-. Find the kinetic energies of π^- and n and the direction of travel of n. $m_\Sigma = 1197$ MeV$/c^2$ $m_\pi = 140$ MeV$/c^2$ and $m_n = 940$ MeV$/c^2$

36. Find the threshold kinetic energy of the following reaction:

    $p + n \rightarrow p + \Sigma^- + K^+$

    $m_p = 938$ MeV$/c^2$, $m_n = 940$ MeV$/c^2$,

    $m_{\Sigma^-} = 1197$ MeV$/c^2$ and $m_{K^+} = 494$MeV$/c^2$ **[1800.4 MeV]**

Section C
### (Answer questions in **one** or **two** pages)

Long answer type questions (Essays)

1. Give an account of various conservation laws in particle reactions of elementary particles.

2. Derive and expression for relativistic threshold kinetic energy of particle reactions of elementary particles.

3. Give a detailed account of standard model

Hints to problems

22. a) $p + p \rightarrow n + p + \pi^+$

    Check charge conservation - conserved check baryon number conservation – conserved. Check spin conservation – conserved.

    b) $p + p \rightarrow p + \Lambda^0 + \Sigma^+$

    B: $1 + 1 = 1 + 1 + 1$ Baryon number not conserved

    c) $e^- + e^+ \rightarrow \mu^+ + \pi^-$

Check charge conservation – not conserved

c) $p + p \rightarrow \pi^+ + K^0 + \Lambda^0$

Charge is conserved   Spin is conserved
Baryon number is conserved   Strangeness is conserved

23. a) $K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$

Charge is conserved   Spin is conserved
Strangeness is not conserved

a) $K^+ \rightarrow \mu^+ + \upsilon$

Strangeness is not conserved

24. a) $\pi^- + p \rightarrow \Lambda^0 + K^0$

Charge is conserved   Spin is conserved
Baryon number is conserved   Strangeness is also conserved

b) $K^- + p \rightarrow \Sigma^+ + \pi^-$

Charge is conserved   Spin is conserved
Baryon number is conserved   Strangeness is conserved

25. π^-, charge $= -1$, spin $= 0$, $S = 0$ and $B = 0$. To satisfy all these $\pi^- \equiv \bar{u}d$.

26. Charge of the particle uus $= \dfrac{2}{3} + \dfrac{2}{3} - \dfrac{1}{3} = 1$ The strangeness $S = -1$, Baryon number

$= +1$ so the particle is Σ^+ .

27. Check baryon number – not conserved check spin – not conserved.

28. For Σ^+, charge $= -e$, B.No $= 1$, strangeness $= -1$

Hence $q_2\, q_2\, q_3$.

29. a) Strong because of life time
    b) Electromagnetic because of photons
    c) Weak because of leptons
    d) Weak because of the lifetime.

30. a) $\Lambda^0 \rightarrow p + K^-$

$m_\Lambda c^2 < m_p c^2 + m_K c^2$

Law of conservation of energy is violated.

b) $\Omega^- \rightarrow \Xi^0 + K^-$

$m_\Omega c^2 < m_\Xi c^2 + m_K c^2$

c) $\Xi^0 \rightarrow \Sigma^0 + \pi^0$

$m_\Xi c^2 < m_\Xi c^2 + m_K c^2$

d) $\mu^- \rightarrow e^- + \gamma$

$L_e : 0 \rightarrow 1 + 0$ not conserved

$L_M : 1 \rightarrow 0 + 0$ not conserved

31. a) $\upsilon_e + p \rightarrow n + e^+$

$L_e : +1 + 0 \rightarrow 0 - 1$ Electron lepton number not conserved

b) $p + p \rightarrow p + n + K^+$

$S : 0 + 0 \rightarrow 0 + 0 + -1$ Strangeness number not conserved

c) $\pi^- + n \rightarrow K^- + \Lambda^0$

$S : 0 + 0 \rightarrow -1 + -1$ Strangeness number is not conserved

32. Total energy, $E = K + mc^2 = 3642 + 1197$

$E = 4839$ MeV

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = 4839 \text{ MeV}$$

From this calculate $\dfrac{v}{c} = 0.9689$

$$\therefore \quad \tau = \frac{\tau_0}{\sqrt{1 - v^2/c^2}} = \frac{1.5 \times 10^{-10}}{\sqrt{1 - 0/9689}} = 6.07 \times 10^{-10} \text{s}$$

$\therefore$ Distance travelled $D = v\tau$

$$D = 0.9689 \, c \times 6.07 \times 10^{-10}$$

$$D = 0.18 \text{m}$$

33. $Q = (m_i - m_f)c^2, \quad m_f = 0$

$Q = m_\pi c^2 = 135$ MeV

34. $K^0 \rightarrow \pi^+ + \pi^-$

From momentum conservation

$$P_{K^0} = P_{\pi^+} + P_{\pi^-}$$

$$0 = p_{\pi^+} + P_{\pi^-}$$

Thus $\left| p_{\pi^+} \right| = \left| P_{\pi^-} \right|$

$$\therefore \quad E_{\pi^+} = E_{\pi^-}$$

From energy conservation

$$E_{K^0} = E_{\pi^+} + E_{\pi^-} = 2E_{\pi^+} \quad (\because \ E_{\pi^+} = E_{\pi^-})$$

$$\therefore \quad E_{\pi^+} = \frac{E_{K^0}}{2} = \frac{498}{2} MeV = 249 \ MeV$$

$$K_{\pi^+} = E_{\pi^+} - m_\pi c^2 = 249 - 140 \ MeV$$

$$= 109 \ MeV.$$

35. $E_\Sigma = K_\Sigma + m_\Sigma c^2 = 250 + 1197 = 1447 \ MeV$

Using $E_\Sigma = \sqrt{(p_\Sigma c)^2 + (m_\Sigma c^2)^2}$

$$\therefore \quad p_\Sigma c = \sqrt{E_\Sigma^2 - (m_\Sigma c^2)^2} = \sqrt{1447^2 - 1197^2}$$

$$p_\Sigma c = 813 \ MeV$$

The conservation of momentum gives x-direction:

$$p_\Sigma = p_n \cos\theta \qquad\qquad .....(1)$$

y-direction

$$O = p_n \sin\theta - p_\pi$$

$$p_\pi = p_n \sin\theta \qquad\qquad ..... (2)$$

From eqn 1 and 2 we get

$$p_n^2 = p_\Sigma^2 + p_\pi^2$$

From conservation of energy

$$E_\Sigma = E_n + E_\pi$$

$$E_\Sigma = \sqrt{p_n^2 c^2 + (m_n c^2)^2} + \sqrt{p_\pi^2 c^2 + (m_\pi c^2)^2}$$
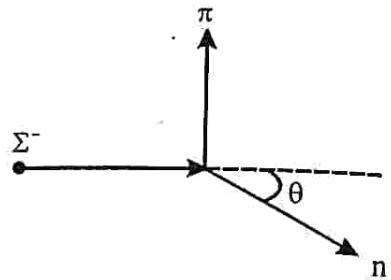
$$\therefore \quad \sqrt{p_\pi^2 c^2 + (m_\pi c^2)^2} = E_\Sigma - \sqrt{p_n^2 c^2 + (m_n c^2)^2}$$

squaring

$$\therefore \quad p_\pi^2 c^2 + (m_\pi c^2)^2 = E_\Sigma^2 + p_n^2 c^2 + (m_n c^2)^2 - 2E_\Sigma \sqrt{p_n^2 c^2 + (m_n c^2)^2}$$

$$\therefore \quad 2E_\Sigma \sqrt{p_n^2 c^2 + (m_n c^2)^2} = E_\Sigma^2 + p_n^2 c^2 + (m_n c^2)^2 - p_\pi^2 c^2 - (m_\pi c^2)^2$$

$$\therefore \quad 2E_\Sigma E_n = E_\Sigma^2 + (p_\Sigma^2 + p_\pi^2)c^2 + (m_n c^2)^2 - p_\pi^2 c^2 - (m_\pi c^2)^2$$

$$\therefore \quad E_n = \frac{E_\Sigma^2 + p_\Sigma^2 c^2 + (m_n c^2)^2 - (m_\pi c^2)^2}{2E_\Sigma}$$

$$E_n = \frac{(1447)^2 + (813)^2 + 940^2 - 140^2}{2 \times 1447}$$

$$E_n = 1250 \text{MeV}$$

$$K_n = E_n - M_n c^2 = 1250 - 940 = 310 \text{MeV}$$

$$E_\pi = E_\Sigma - E_n = 1447 - 1250 = 197 \text{MeV}$$
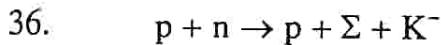
$$K_\pi = E_\pi - m_\pi c^2 = 197 - 140 = 57 \text{MeV}$$

$$cp_n = \sqrt{E_n^2 - (m_n c^2)^2} = \sqrt{1250^2 - 940^2} = 824 \text{ MeV}$$

From eqn. 1 we get

$$\cos\theta = \frac{p_\Sigma}{p_n} = \frac{cp_\Sigma}{cp_n} = \frac{813}{824} = 0.9866$$

$$\theta = 9.37°$$

36. $p + n \rightarrow p + \Sigma + K^-$

$$Q = (m_i - m_f)c^2$$

$$Q = (m_p + m_n - m_p - m_\Sigma - m_K)c^2$$

$$Q = (m_n - m_\Sigma - m_K)c^2$$

$$Q = 940 - 1197 - 494 \text{ MeV}$$

$$Q = -751 \text{ MeV}$$

Threshold kinetic energy

$$K_{th} = \frac{-Q(m_p + m_n + m_p + m_\Sigma + m_K)}{2m_n}$$

$$K_{th} = -Q \frac{(2m_p + m_n + m_\Sigma + m_K)c^2}{2m_n c^2}$$

$$K_{th} = --751 \frac{(2 \times 938 + 940 + 1197 + 494)}{2 \times 940}$$

$$K_{th} = 1800.4 \text{ MeV}$$